Original Article doi: 10.1590/0100-512X2025n16008es

DESCARTES ON THE ROLE OF DIAGRAMS AND SYMBOLS IN MATHEMATICS[1]

DESCARTES SOBRE O PAPEL DOS DIAGRAMAS E SÍMBOLOS NA MATEMÁTICA

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ABSTRACT *Descartes seems to have held to the end of his life that imagination plays a prominent role in mathematics. In distinguishing sharply between images and ideas in the* Meditations*, he is adamant in stating that we do not need images in order to have mathematical ideas, and that there is a purely intellectual conception of extension and even presumably extended objects, existing or not. However, he seems to advocate that imagination does make these ideas better. The main goal of the present paper is to analyze, specially by means of Rules XIV-XVI and the* Discourse on Method*, how this auxiliary role played by imagination can be found in the use of diagrams and mathematical symbolism as an aid for mathematical invention. As I will argue, while Descartes seems to have thought that (ordinary) language is a source of error, prejudice, and obscurity, he also thought that a good use of signs in*

1 Article submitted on: 21/02/2025. Accepted on: 30/05/2025.

mathematics provides our ideas with distinctness, as also would the remote possibility of an idealized universal philosophical language.

Keywords: *Descartes. Mathematics. Symbolism. Diagrams. Imagination.*

RESUMO *Descartes parece ter defendido até o final de sua vida que a imaginação exerce um papel fundamental na matemática. Como tornam patente tanto a representação puramente intelectual da cera quanto sua distinção entre ideia e imagem nas* Meditações*, o filósofo admitia que não dependemos de imagens para ter ideias de objetos extensos, existentes ou não. Todavia, ele também afirma que a imaginação torna o nosso conhecimento da extensão, da figura e do movimento "muito melhor" (*beaucoup mieux*). O principal objetivo do presente artigo é examinar, particularmente por meio das Regras XIV-XVI e do* Discurso do Método*, como esse papel acessório exercido pela imaginação pode ser encontrado no uso de diagramas e simbolismo para auxiliar a invenção matemática. Defenderemos que, ao passo que Descartes considerava a linguagem (ordinária) como uma fonte de erro, preconceito e obscuridade, ele também acreditava que um uso adequado de sinais matemáticos seria capaz de proporcionar distinção às nossas ideias, como também o faria (a possibilidade remota de) uma linguagem filosófica universal.*

Palavras-chave: *Descartes. Matemática. Simbolismo. Diagrama. Imaginação.*

I - Descartes on language

Descartes is not typically regarded as having a particularly positive view of language or speech (*loquela*). For one thing, in a reply to an objection by Hobbes, he firmly asserts that in reasoning "there is a coupling not of names, but of the things signified by the names, and I am amazed that anyone can think the contrary" (AT VII, p. 178). Like most early modern philosophers, he also embraces a traditional view according to which the meaning of words (and maybe other expressions) depends entirely on their correspondent ideas in the mind: "we cannot express anything by our words, when we understand what we say, without it being certain that we have in us the idea of the thing which is signified by our words" (AT III, p. 393).

This view is often referred to as 'mentalism' and it can be traced back at least to Aristotle's *De Interpretatione*, where he famously states that "spoken sounds (τὰ ἐν τῇ φωνῇ) are symbols of affections in the soul (τῶν ἐν τῇ ψυχῇ παθημάτων), and written marks (γραφόμενα) symbols of spoken sounds" (16a3). In Descartes' philosophy overall and in his views on the proper acquisition of knowledge, it is certainly thoughts and ideas—not language—that take precedence. He says, for instance, in his *Meditations On First Philosophy* (1641) that "one who wants to achieve knowledge above the ordinary level (*supra vulgus*) should feel ashamed at having taken ordinary ways of talking (*formis loquendi quas vulgus invenit*) as a basis for doubt" (AT VII, p. 32).

This is because, at least as far as Descartes' later thought is concerned, language is—much like Bacon's *idola fori*[2]—a source of error and prejudice. The view is already prevalent in the *Meditations*: "Although I am thinking about these matters within myself, silently and without speaking, nonetheless the actual words bring me up short, and I am almost tricked by ordinary ways of talking (*ab ipso usu loquendi*)" (AT VII, p. 31-32). However, a more comprehensive account appears in his *Principles of Philosophy* (1644), where he identifies speech as the fourth cause of error (the other three being respectively the prejudices of our childhood, the fact that we are unable to forget these prejudices, and the fact that we grow accustomed to judging things based on preconceived notion rather than present perception):

> [...] because of the use of speech, we attach all our concepts to words by which we express them, and do not commit them to memory except along with these words. And since we afterwards more easily remember the words than the things; we scarcely ever have a concept of any thing so distinct that we separate it from all conception of the words; and the thoughts of almost all men are occupied with words more than with things. Thus men very often give their assent to words which they have not understood; because they think that they formerly understood them or learned them from others who understood them correctly. (AT VIIIA, pp. 37–38)

According to Descartes, by attending to words rather than the concepts they express we make ourselves prone to making judgment without proper understanding of the matter at hand. Despite this fundamental deficiency, he acknowledges in the *Discourse on the Method* (1637) that a proper use of language is one of two criteria for telling 'real human beings' (*vrais hommes*) apart from both animals or beasts and automata, for they "would never be able to use words or other signs by composing them as we do to declare our thoughts to others" (AT VI, p. 56, the other being the ability to 'operate in all sorts of situations' due to the versatility of reason). Moreover, Descartes seems to believe that the shortcomings of language can somehow be at least mitigated.

2 See *Novum Organum, XLIII.*

In a letter to Mersenne dated November 20, 1629, Descartes addresses a proposal submitted by his correspondent of an artificial universal language with a regular grammar that could be quickly learned. His overall stance on the subject is notably skeptical, doubting both the feasibility and usefulness of such a project. He highlights several drawbacks both in matters of lexicon and grammar. Nonetheless, he remains reasonably sympathetic and open to the broader idea of a similar endeavor:

> This can be accomplished using order, that is, by establishing an order among all the thoughts that can enter the human spirit in the same way that there is an order naturally established among the numbers. We can learn to name all the numbers up to infinity in a single day and to write them in an unknown tongue, using an infinite number of different words; in the same way, we could do this for all other words necessary to express all the other things that fall on the spirits of men. If this were discovered, I have no doubt that the language would soon spread around the world (AT I, p. 80).

Descartes appears to admit the possibility of a language of this kind: “I hold that this language is possible and that the knowledge on which it depends can be discovered” (AT I, p. 81). There is, however, a major catch. According to him, the enumeration of our thoughts and the ensuing invention of this language depend on an analysis of these thoughts and ultimately “on the true philosophy” (Ibid.). He also suggests that its implementation would require someone to explain the nature of the “simple ideas [...] out of which all thoughts are composed” (Ibid.) in such a way that it would be generally accepted. For this reason, Descartes does not seem to expect this language to be developed or implemented anytime soon, since, as he puts it, “the whole world would have to be a terrestrial paradise, something good to suggest only for the realm of novels (*les pays des romans*)” (AT I, p. 82).

If it were devised, however, this language would seem to eradicate the main issue with language that Descartes later points out in his *Principles of Philosophy*, as we have seen. As things stand, “almost all our words have confused meaning and people’s minds are so accustomed to this over a long time that there is almost nothing they understand perfectly” (AT I, p. 81). On the other hand, the main advantage of this idealized language would consist precisely in its ability to assist judgment “by representing all things so distinctly that it would be almost impossible for one to be deceived” (Ibid.). This would allow “peasants to judge the truth of things better than philosophers do now” (AT I, p. 82).

II - Mathematics and imagination

What is perhaps most striking about Descartes' description of his version of this idealized language is that he seems to acknowledge that its most attractive features are already present in our ordinary numerical expressions. As we have seen, this intrinsic order of our thoughts is compared to the "order naturally established among the numbers" and he remarks that "we can learn to name all the numbers up to infinity in a single day and to write them in an unknown tongue, using an infinite number of different words" (AT I, p. 80). Here it seems like Descartes is recognizing a partial accomplishment in our numerical expressions in this regard.

He does not merely point out that numbers have a natural order; he also acknowledges that we have developed adequate means of expressing this order—something we can only dream of achieving with the full range of our ideas or thoughts. Doing so would result precisely in (the remote possibility of) the establishment of the idealized language we have been discussing. By modeling his idea of a philosophically perfect language on numerical expressions, Descartes gives us a glimpse of just how much he was concerned with the proper use of signs in mathematics.

One might assume that Descartes' critique of ordinary language could be directly transferred to the use of signs in mathematics, suggesting that mathematics should be adequately approached solely as an entirely intellectual enterprise. I believe this is mistaken. In this paper, I will argue that, according to Descartes, diagrams and symbolism play a crucial role in mathematics under the general title of the faculty of imagination. Although there are good and bad uses of such signs, as we will discuss, Descartes views them as pivotal tools in mathematical *invention*—not only in its communication. The primary function of these signs is to aid one's thinking, rather than merely conveying results, conjectures, and proofs to others in a precise manner. In fact, as a practicing mathematician, Descartes was consistently concerned with notation and its improvement.

So much so that his *Geometry* (1637) is often regarded as one of the first mathematical works whose notation is readable to those versed only in modern notation. As a matter of fact, apart from a few minor details—such as his introduction of the symbol "∞" (which, after Wallis, is nowadays used to signify infinity) for identity, instead of Robert Recorde's "="—Descartes' algebraic notation is pretty much the same as ours. He is responsible for introducing (a) the superscript notation for exponentiation using Arabic numerals as exponents "2^2", (b) the use of letters from the end of the alphabet for variables and unknowns "x, y, z", (c) the radical symbol "√" for square root with the vinculum "‾‾‾" above the radicand to indicate the scope of the operation, and (d) the vertical line notation for proportion "a|b||c|d", among other contributions[3].

Both diagrams and mathematical symbols, as we will explore in more detail later, serve as aids provided by the faculty of imagination. In the *Second Meditation*, Descartes defines imagining as "nothing other than contemplating the shape (*figuram*) or image of a bodily thing" (AT VII, p. 28)[4]. He also writes in a letter to Henry More that "it is indeed true that nothing falls within the scope of the imagination (*sub imaginatione cadit*) without being in some way extended" (AT V, p. 270). As we have suggested, Descartes viewed imagination as playing a pivotal role in mathematical thinking. I believe this will become clear through our analysis of *Rules XIV-XVI* and the correspondent passages in the *Discourse on the Method*. However, I think there is sufficient evidence to establish that this is not a transitory early view, which was later abandoned by Descartes.

In a letter to Mersenne from November 13, 1639, Descartes writes that "the part of the spirit *which most helps mathematics, namely, imagination*, is more of a hindrance than a help in metaphysical speculation" (emphasis added, AT II, p. 622). As late as 1643, in a letter to Elisabeth of Bohemia, he states that "the study of mathematics, which exercises mainly the imagination in the consideration of shapes and motions, accustoms us to form very distinct notions of body" (AT III, p. 692). However, as the citation from the letter to Mersenne suggests, this auxiliary role of imagination does not extend in the same way to metaphysics. Descartes even asserts, for instance, that God and the soul "are not imaginable, but only intelligible" (AT V, p. 270), since they are not extended. Similarly, in physics—unlike in mathematics—he says that "there is no place for imagination" (AT V, p. 177).

The case of metaphysics shows that there are things we conceive independently of imagination, as they are not extended. The case of physics, on the other hand, shows that there is a conception of extended things that is not aided by the imagination. According to Descartes, the object of physics and that of mathematics are the same, but in physics, we conceive it as actually existing, while in mathematics, we conceive it as merely possible:

3 See CAJORI (1993).

4 He sometimes uses imagination in a stricter sense: "The difference between sense-perception and imagination is thus really just this, that in sense-perception the images are imprinted (*pingantur*) by external objects which are actually present, whilst in imagination the images are imprinted by the mind without any external objects, and with the windows shut, as it were." (AT V, p. 162)

> This object [of mathematics] has a true and real nature, just as much as the object of physics itself. The only difference is that physics considers its object not just as a true and real entity, but also as something actually and specifically existing (*tanquam actu et qua tale existens*). Mathematics, on the other hand, considers its object merely as possible, i.e. as something which does not actually exist in space but is capable of so doing. (AT V, p. 160)

Descartes' famous example of the wax in the *Second Meditation* also establishes that there is in fact a conception of extended things without any appeal to the imagination: "So I am left with no alternative, but to accept that I am not at all imagining what this wax is, I am perceiving it with my mind alone" (AT VII, p. 31). He later adds: "Certainly it is the same wax I see, touch, and imagine [...]. But yet—and this is important—the perception of it is not sight, touch, or imagination, and never was, although it seemed to be so at first: it is an inspection by the mind alone [...]" (Ibid.). Moreover, in the *Sixth Meditations* and especially in his replies to Hobbes' objections, Descartes is adamant about distinguishing between images and ideas, as well as between imagination and pure intellection:

> [...] when I imagine a triangle, not only do I understand it to be a shape enclosed by three lines, but at the same time, with the eye of the mind (*acie mentis*), I contemplate the three lines as present, and this is what I call imagining. But if, on the other hand, I wish to think of a chiliogon, I do indeed understand that this is a shape consisting of a thousand sides, no less clearly than I understand that the triangle consists of three: but I do not imagine the thousand sides in the same way, that is, contemplate them as present. And although at the time, because I am accustomed always to imagine something whenever I am thinking of bodily things, I may perhaps picture some figure to myself in a confused fashion, it is quite clear that this is not a chiliogon, because it is not at all different from the picture I would also form in my mind if I were thinking about a myriogon, or some other many-sided figure. Nor is it of any assistance in recognizing the properties by which the chiliogon differs from other polygons [...] and here I observe very plainly that I need to make a particular effort of the soul (*animi contentione*) in order to imagine, that I do not make when understanding. This further effort of the soul clearly indicates the difference between imagination and pure intellection. (AT VII, pṕ. 72-3)

This passage is important because the examples Descartes provides are all geometric. If imagination is not a necessary condition for conceiving extended things—even in mathematics—and sometimes seems to fall short of pure intellection, as with the chiliogon and the myriogon, then what exactly is the auxiliary role that imagination plays in this science? A first clue can be found in the same letter to Elisabeth, where he writes: "The soul is conceived only by the pure intellect; body (i.e. extension, shapes and motions) can likewise be known by the intellect alone, *but much better* (*beaucoup mieux*) *by the intellect aided by the imagination*" (emphasis added, AT III, p. 691).

A more detailed answer, however, requires a thorough analysis of *Rules XIV-XVi* and their corresponding passages in the *Discourse on the Method*, which will be presented in Section IV of this paper. To do so, however, we would benefit from a previous presentation of Descartes' views on the object of mathematics as a science and how it is divided.

III - Descartes on the object and division of mathematics

As mentioned in the previous section, Descartes viewed mathematics and physics as having the same object only considered in different ways. This object is extension or body—that which is extended[5]—as such. In the *Fifth Meditation*, he states that corporeal nature is the subject matter of pure mathematics (AT VII, p. 71). In the French translation by de Luyne, he adds, however, two qualifications: (a) this nature is taken only insofar as it is subject to geometrical demonstrations, and (b) it is conceived without requiring actual existence. In the *Sixth Meditation*, Descartes remarks that this corporeal nature, insofar as it is the subject matter of pure mathematics, excludes such things as 'colours, sounds, tastes, pain, and so on' (AT VII, p. 74). He also notes that while pure mathematics does not concern itself with the actual existence of its objects, this does not take anything away from its certainty—at least once God's existence is established (AT VII, p. 64).

Everything we demonstrate about mathematical entities is true, even if they do not exist. In fact, Descartes holds that while geometrical entities might not exist in reality, their existence is certainly possible. In his reply to Gassendi's objection, where the latter argues that entities such as "points, lines, surfaces, and indivisible figures which are composed of elements and yet remain indivisible," cannot exist in reality (AT VII, p. 329), Descartes answers that "the world could undoubtedly contain figures such as those the geometers study" (AT VII, p. 380-1). While no extended substance can lack length, breadth, or depth—since that is what extension is—geometrical objects like points, lines, and surfaces are instead considered rather as "boundaries (*termini*) within which an extended substance is contained" (AT VII, p. 381).

5 See AT X, p. 444.

Another strategy Descartes employs is to argue that points, lines, and surfaces are bodies in which either breadth, depth, or extension altogether is not considered. He also talks of them as modes of extension:

> But the name 'surface' is used in two senses by mathematicians. In one sense they use the term of a body whose length and breadth alone is under consideration and which is regarded quite apart from any depth it may have, even though the possession of some degree, of depth is not ruled out; alternatively, they use the term simply for a mode of body, in which case all depth is completely denied (AT VII, p. 433).

In the *Rules*, Descartes identifies different types of what he then called 'simple natures': intellectual or spiritual, corporeal or material (extension, shape, motion), and common ones—those which are, as the name suggests, common to both mind and body (duration, unity, existence). The nature of mathematical objects are generally conceived as somehow composed of material and common simple natures (AT X, 422). Extension, which consists in length, breadth, and depth, is the sole nature or the principal attribute of body or extended substance. Later classifications include position and divisibility of component parts on the side of material natures, while substance, order, and number appear as common natures.

While extension constitutes the nature of body (AT VIII, p. 42), shape, motion, and the like are regarded as modes of extension (AT VIII, p. 29)[6]. However, all material simple natures fall within the domain of pure mathematics. The inclusion of motion as a subject of *pure* mathematics might surprise those unfamiliar with 17th-century mathematical practice. Yet, the use of motion in both definitions and proofs was a mark of the mathematics of the period and was closely related to foundational debates regarding its status as a science[7]. In response to Ciermans' comments on his *Geometry*, Descartes states:

> I detect in you a singular courtesy (*humanitatem*) [...] when you say that the few things I wrote about geometry deserve to be called pure mathematics; for I never explained anything that properly pertains to arithmetic nor did I resolve any of the questions in which one considers both order and measure at the same time, examples of which are given in Diophantus. But, in addition, I did nothing with respect to motion, in the examination of which, however, pure Mathematics*, at least as I practised it, is chiefly engaged.* (emphasis added, AT II, pp. 70-1)

Furthermore, Descartes holds that extension and quantity are fundamentally the same, their distinction being purely conceptual. Thus, despite seeing

6 Descartes also mentions position and divisibility of component parts.
7 See MANCOSU (1996).

mathematics as a science of extension (considered in a particular way), he remains, in a sense, aligned with the classical definition of mathematics as the general science of quantity, geometry being the science of continuous quantity (magnitude, μέγεθος) and arithmetic of discrete quantity (number, ἀριθμός)[8]. Although Descartes does not always adhere strictly to this terminology[9], he maintains the general outlook of the distinction. He explicitly states his thesis of the identity between extension and quantity—both in the case of magnitude and number—in *Principles* 2, §8:

> There is no real (*in re*) difference between quantity and the extended substance; the difference is merely a conceptual one (*ex parte nostri conceptus*), like that between number and the thing which is numbered. We can, for example, consider the entire nature of the corporeal substance which occupies a space of ten feet without attending to the specific measurement; for we understand this nature to be exactly the same in any part of the space as in the whole space. And, conversely, we can think of the number ten, or the continuous quantity ten feet, without attending to this determinate substance. For the concept of the number ten is exactly the same irrespective of whether it is referred to this measurement of ten feet or to anything else; and as for the continuous quantity ten feet, although this is unintelligible without some extended substance of which it is the quantity, it can be understood apart from this determinate substance. In reality (*in re*), however, it is impossible to take even the smallest fraction from the quantity or extension without also removing just as much from the substance; and conversely, it is impossible to remove the smallest amount from the substance without taking away just as much from the quantity or extension (AT VIII, pp. 44-5).

A similar framework appears in the well-known *Rule IV*, where Descartes asserts that mathematics concerns itself with questions of order or measure. The general science that deals exclusively with these two types of relations, regardless of the subject matter, is what he calls universal mathematics (*mathesis universalis*). In the same rule, Descartes also hints at the disciplines that fall under the category of *mixed* mathematics, mentioning astronomy, music, optics, and mechanics (AT X, p. 377), in a loose continuity with the Pythagorean distinction.

8 Descartes refers to algebra as a sort of arithmetic (AT X, p. 373).

9 He seems to often use "magnitude" as the most general term, referring to that which admits differences of more and less (AT X, p. 440), while he characterizes quantity as 'continuous magnitude" (AT X, p. 452, see also AT VIII, pp. 44-5). However, he also frequently uses "number" to encompass all these cases. For instance, in his *Geometry*, he talks about "surd numbers" (AT VI, p. 452),—an older term for irrational numbers, traditionally considered magnitudes, i.e., a continuous quantities, with no numerical counterpart. He also speaks of true (positive, AT VI, p. 446), false (negative, AT VI, p. 445), and imaginary numbers (AT VI, p. 453). See also AT X, p. 385.

It is fairly straightforward to see how the general characterization of mathematics as a science of extension applies to the case of geometry. Regarding its subject matter, Descartes states in the *Discourse on the Method*:

> After that, wishing to seek other truths, I considered the object studied by geometers. I conceived of this as a continuous body, or a space indefinitely extended in length, breadth and height or depth, and divisible into different parts which may have various shapes (*figures*) and magnitudes (*grandeurs*), and may be moved or transposed in every way: for all this is assumed by geometers in their object of study (AT VI, p. 36).

However, the connection between arithmetic and the characterization of mathematics as a science is a lot more complicated. After all, it is reasonable to think that we do not only count extended things, but also unextended things, i.e, we count cards and shells but also theorems. How, then, can a science concerned with number, discrete quantity, multitude, or order be considered a science of corporeal nature or extension? For one, Descartes did not think of numbers as self-standing abstract entities. Instead, he regards them as (non-Platonic) universals: "[...] we should not regard order or number as anything diverse from the things which are ordered and numbered" (AT VIII, p. 26). Rather "[...] number, when it is considered simply in the abstract or in general, and not in any created things, is merely a mode of thinking" (AV VIII, 27).

This, however, does not solve our problem. Saying that numbers are not separate from the things numbered is not to say that these things are extended. Does Descartes view number as something attributed exclusively to extended things? There is strong evidence to the contrary. As we've seen, unity[10], number, and order are listed among the *common* natures, which suggests that unity and number can also apply to immaterial or unextended beings. Descartes might be seen as making a move somewhat similar to that of Thomas Aquinas. Not only did Aquinas think that numbers were mind-dependent concepts, he also thought of "one" and "number"[11] in the strict mathematical sense as applying only to bodies, while acknowledging an analogous sense in which they could be attributed to all kinds of things.

This division allows Aquinas to deal with metaphysical issues, such as the problem of the divine Trinity and the multitude of angels. In fact, Aquinas defines this mathematical sense of "one" and "number", which pertains to the category of quantity, as the order of parts in a whole whose parts are

10 Of course, Descartes may be referring to unity as a transcendental in the Scholastic sense—but more on that later.

11 Since Greek antiquity, it was common to distinguish between one, unit, or unity and number, as number was often defined as a collection or aggregate of units.

disconnected or materially divided or an aggregate of materially divided units. There is, however, a transcendental sense of "one" and "many", which pertains to all categories[12]. Descartes might also be seen as thinking of number in a twofold way: one sense applies to body or extension and pertains strictly to mathematics, while the other applies to all kinds of entities, making it a common nature. Another possibility is that although there is only one notion of number, arithmetic studies it only insofar as it applies to extended things.

Descartes is, of course, also famous for holding several controversial theses regarding the nature of mathematics, such as the innateness[13] of at least some mathematical concepts or propositions, the free creation of the eternal truths by God, and the idea that there are grounds for doubting the whole of mathematics unless one is provided with knowledge of God's existence. However, all of these topics are beyond the scope of the present paper. Having outlined Descartes' views on the nature and structure of pure mathematics, we will now, in the next section, examine his views on the role of signs therein.

IV - The role of diagrams and symbols in the *Rules* and in the *Discourse*

The *Regulae ad Directionem Ingenii* is an unfinished early work that Descartes appears to have been working on around 1628. The existence of different versions of the text—the recently discovered early Cambridge manuscript, Leibniz's copy, and the later Amsterdam printing—suggests that he revisited the work later. Much like his later *Discourse,* the *Rules* is largely a methodological treatise. It is primarily concerned with establishing a method to direct the *ingenium* towards solid and true judgments through certain and indubitable cognition, i.e., science. According to the *Rules*, there are, as of yet, only two sciences in this strict sense of the word: arithmetic and geometry. Additionally, there are only two operations by which we can acquire this kind of cognition: intuition (*intuitus*) or deduction.

12 See SVOBODA & SOUSEDIK (2014).

13 In fact, Descartes seems to think that the innateness of our geometrical ideas is a condition for our use of diagrams in geometry (AT VII, p. 382) He does mention that we form ideas of numbers and geometrical figures when encountering instances of them (AT VIII, pp. 27-8) The difficulty can be overcome, however, if we keep in mind that Descartes' doctrine of innateness consists solely in the thesis that there are "certain thoughts within me [...] which came solely from the faculty of thinking within me" (AT VIII, p. 358), which can be triggered by our experiences. He insists that "there is nothing in our ideas which is not innate to the mind or the faculty of thinking, with the sole exception of those circumstances which relate to experience". These ideas "always exist within us *potentially* (*potentia*), for to exist in some faculty is not to exist actually, but merely potentially, since the term 'faculty' denotes nothing but a potentiality" (AT VIII, p. 361).

Descartes defines intuition as "the conception (*conceptum*) of a pure and attentive mind (*mentis purae et attentae*), which is so easy and distinct that there can be no room for doubt about what we are understanding," to which he gives a couple examples, two of which are geometrical: "that a triangle is bounded by just three lines, and a sphere by a single surface" (AT X, p. 368). On the other hand, he characterizes deduction as the "mode of knowing (*cognoscendi modum*) [...] by which we understand everything that necessarily follows from some other things which are known with certainty," to which he adds "provided they are deduced from true and known principles through a continuous and nowhere interrupted movement of thought in which each individual thing is perspicuously intuited" (AT X, p. 369).

There is a case in which a deduction counts as a single intuition: "[...] whatever we have immediately deduced from other things, if the inference (*illatio*) is evident, it already comes under the heading of true intuition" (AT X, p. 389). In all remaining cases, however, certainty is not guaranteed by "present evidence" but is preserved by the links of the (uninterrupted) deduction chain. Due to its successive and mediate character, as opposed to intuition's immediacy, deduction gets its certainty from memory and "a continuous movement of thought is needed to make good on any weakness (*infirmitati*) of memory" (AT X, p. 387).

This "continuous movement" is explained as consisting of "simultaneously intuiting one relation and passing on to the next, until I have learnt to pass from the first to the last so swiftly that memory is left with practically no role to play, and I seem to intuit the whole thing at once" (AT X, p. 388). However, when the multiplicity of steps does not allow for such a continuous movement of thought, we must resort to an auxiliary procedure of surveying or checking each step in the deduction. Descartes refers to this procedure as induction or enumeration[14], which seems to be more vulnerable to the shortcomings of our memory.

The *Rules* were originally conceived to be divided into three parts, each one containing twelve rules. The first being on philosophy, the second on mathematics (which remained incomplete), and the third on natural philosophy (which was never written). *Rule XII*, i.e., the twelfth rule of the first part, which introduces the following section on mathematics, is concerned specifically with how and to what extent the imagination, the senses, and memory aid the intellect, the latter being the only faculty able to perceive the truth. Remarking

14 This is in fact only one sense of enumeration in the *Rules*, see Beck, 1952, 143.

on the aid of the imagination and the senses on the intellect, Descartes says that these faculties do not aid the intellect in dealing with entities that are neither corporeal nor similar to it, but rather hinder it. However, concerning the body or what is similar to it the very opposite is the case:

> If the intellect is dealing with these matters in which nothing is corporeal or similar to the corporeal, then it cannot be helped by these faculties. Rather, in order that it might not be hindered by them, the senses have to be kept back, and the imagination has to be divested, as far as possible, of every distinct impression. On the other hand, if the intellect proposes to examine something which can be referred to the body, then the idea of that thing has to be formed as distinctly as possible in the imagination (AT X, pp. 416-7).

Here we see an idea that is later mentioned in Descartes' correspondence with Elisabeth of Bohemia: the imagination is responsible for providing us with (very) distinct ideas or notions of the corporeal. It is by providing such distinctness to these ideas that the imagination helps make "the body (i.e., extensions, shape, and motion)" (much) better known[15]. Anticipating the subject of *Rules XIV-XVI*, Descartes suggests that, to do this in the most efficient way, we should exhibit instances of those ideas to our external senses or, rather, abbreviations of them:

> And in order to accomplish this more conveniently, the thing itself, which this idea will represent, has to be displayed to the external senses. Nor can many things aid the intellect in distinctly intuiting individual things. Rather, in order that it might deduce one thing from many things gathered together, which must often be done, one has to discard, from the ideas of the things, whatever does not require one's present attention, in order that what remains may be more easily retained in the memory. In the same way, it is not so much the things themselves, as rather certain abbreviated representations of them (*compendiosae illam quaedam figurae*), which are to be presented to the external senses. And, the more compact these abbreviated representations are, the more convenient they are, provided that they suffice to prevent lapses of memory (AT X, p. 417).

This passage introduces the requirement, highlighted in *Rule XVI*, of notational economy, brevity, or compactness. This requirement helps us prevent a profusion of elements in the imagination so as to not be overwhelmed by

15 Nolan (2005) has already suggested the idea that the imagination has an overall role in promoting clear and distinct conceptions of corporeal nature. However, he provides no analysis of the specific heuristic role of diagrams and symbols in mathematical invention as presented in Descartes' early methodological treatises. In this paper, I aim to show that they already performed such a role in Descartes' earlier writings—including the *Discourse on the Method*, which has been published—and to examine, based on Descartes' direct comments on the subject, how a proper use of their heuristic features helps to promote distinctness, in contrast to the obscurity inherent in ordinary language.

them, enabling us to grasp these elements and their relations "in one fell swoop" as much as possible. Compact signs play a crucial role in preventing irrelevant and unnecessary information from creeping in unnoticed. This brachigraphic feature also supports their mnemonic function, which, as we will explore later in the same rule, is also very important in deductions—either by surveying their steps one by one through enumeration or assisting one to attain that "continuous movement" of thought. This particular application of the imagination is suggested in *Rule VII*:

> Thus, e.g. if I have first found out by diverse operations what the relation (*habitudo*) is between the magnitudes A and B, then what between B and C, between C and D, and finally between D and E, that does not entail my seeing what the relation is between A and E, nor can the truths previously learnt give me a precise understanding of it unless I recall them all. *To remedy this I would run them over several times, keeping the imagination moving continuously in such a way that I would be simultaneously intuiting one relation and passing on to the next*, until I have learnt to pass from the first to the last so swiftly that memory is left with practically no role to play, and I seem to intuit the whole thing at once. For in this way, while aiding the memory, the slowness of the *ingenium* is also corrected, and its capacity is, in a certain manner, expanded. (emphasis added, AT X, pp. 387-8).

In *Rule IV*, Descartes observes that if it were not for this profusion of symbols or 'multiplicity of numbers and incomprehensible figures which overwhelm it', algebra would be the same as the ancient's hidden art of true mathematics—traces of which can be seen in the work of mathematicians such as Pappus and Diophantus. Instead, this symbolic excess hinders the attainment of such mathematics, as it obstructs "perspicuity and easiness":

> In the present age some very gifted men have tried to revive this art, for it seems to me to be none other than that which goes by the outlandish name of 'algebra'—or at least it would be if algebra were divested of the multiplicity of numbers and incomprehensible (*inexplicabilibus*) figures which overwhelm it and instead possessed that abundance of perspicuity and easiness (*facilitas*) which I believe the true mathematics ought to have (AT X, p. 377).

This issue, however, is not limited to algebra. It also appears in geometrical analysis and in a diagrammatic use of signs, which can similarly strain the imagination:

> As for ancient geometrical analysis and modern algebra, even apart from the fact that they deal only in highly abstract matters that seem to have no practical application, the former is so closely tied to the consideration of figures that it is unable to exercise the intellect without greatly tiring the imagination, while in the latter case one is so much a slave to certain rules and symbols (*chiffres*) that it has been turned into a confused

> and obscure art that bewilders the spirit (*embarasse l'esprit*) instead of being a science that cultivates it (AT VI, pp. 17-8).

This passage from *Discourse on the Method* illustrates not only that the problem of a profusive use of signs is not exclusive to algebra but also highlights another recurrent issue with the use of symbols in this branch of mathematics: it often makes one susceptible to becoming a "slave to certain rules and symbols," turning the art into something confused and obscure rather than clear and distinct, ultimately "bewildering the mind." This brings us to another important feature of the proper use of signs in mathematics which is closely tied to Descartes' critique of language: his rejection of the practice of focusing more on signs (symbols, figures, or words) than on the things they signify or stand for. Even in mathematics, when we lose sight of the meaning behind the signs we are employing, we fall back into a state of obscurity and confusion.

As Rezende (2023, p. 41) points out, this can be seen as an early criticism of a sort of symbolic or blind thought, a concept later developed by Leibniz. Descartes, of course, does not argue that it is impossible to think purely in terms of signs; rather, he warns against doing so. He perfectly acknowledges that we can conduct our examinations based solely on signs, ignoring the thoughts they signify. However, in accessing Hobbes' objections, Descartes denies that this is an essential aspect of thinking. He argues that relying on signs alone can easily lead us astray. Instead, Descartes emphasizes that when dealing with signs, one should be guided by the intellect through a consideration of what these signs stand for.

In much the same spirit, he says that "there is really nothing more futile (*inanius*) than so busying ourselves with bare numbers (*nudos numeros*) and imaginary figures that we seem to rest content in the knowledge of such trifles (*nugarum*)" (AT X, p. 375). Additionally, in a famous and often cited passage from the *Discourse on the Method*, which includes *The Geometry* as one of its appendices, Descartes highlights a kind of back and forth correction between algebra and geometrical analysis. While (concise) algebraic symbols aid memory and prevent a profusion of figures from exhausting or overwhelming the imagination, geometrical figures themselves—being the most effective tool for considering proportions separately—help trace these symbols back to their original meanings:

> Then, having noted that, in order to know them [proportions], I would sometimes need to think about them separately, and sometimes only bear them in mind, or consider many together, I came to the view that, in order to consider these proportions best

> separately, I had to suppose them to hold between lines, because I found nothing simpler nor more capable of being distinctly represented to my imagination and to my senses. But for the purpose of retaining them in my memory, or grasping several together, it was necessary for me to designate them by the briefest possible symbols (*les expliquasse par quelques chiffres, les plus courta possible*); by this means I would borrow what was best from geometrical analysis and algebra, and would correct all the defects of the one by the other (AT VI, p. 20).

In summary, there is both a good and a bad use of signs. The good use prevents profusion, aids memory, preserves the meaning of signs, and helps provide our mathematical ideas with distinctness. The bad use, on the other hand, fails to do so. Descartes also recognizes a misuse of the faculty of imagination in general in mathematics (AT X, p. 375).[16] He is consistent in criticizing the use of imagination in mathematics when it lacks the guidance of a proper method and thorough reasoning, although he seems to acknowledge that it may occasionally be effective (AT V, p. 177).

In *Rule XIV*, Descartes continues his investigation of how imagination can assist the intellect. He establishes that we only arrive precisely at truth by some kind of comparison—unless when we are contemplating a single thing in isolation. Comparisons are only straightforward when the objects in question equally share the same nature. In many cases, however, their nature is shared only in a qualified manner, i.e., according to some relations and proportions. These cases require further preparation.

Descartes asserts that "the chief part of human industry" lies in reducing these proportions to "a clear equality between what is sought and something which is known" (AT X, p. 441). He adds that this is only applicable to magnitudes, i.e., what admits differences of more or less[17]. In fact, Descartes' conception of mathematics, as presented in the *Discourse*, is purely relational, as it is exclusively based on these different relations (*habitudines*)[18] or proportions:

> Yet I did not, for all that, intend to study all those particular branches of knowledge which habitually go under the name of mathematics; I saw that, although their objects were different, they nevertheless all concurred insofar as they only took into consideration the

16 Compare with AT X, p. 375: "But at that time, no writers who fully satisfied me happened to come into my hands in either field. For indeed, I read many things in them concerning numbers, which, after careful calculation (*subductis rationibus*), I found to be true. However, regarding figures, they presented many things as if to the eyes themselves and drew conclusions from certain consequences. Yet, why these things were so and how they were discovered did not seem to be sufficiently shown (*ostendere*) to the mind itself."

17 See note 10.

18 The term *habitudo* traditionally translates the Greek σχέσις into Latin. A ratio, for instance, is characterized as a kind of *habitudo/σχέσις* in Euclid's *Elements* (Book 5).

> different relations or proportions to be found among these objects, and I came to think that it was best for me to examine only these proportions in general (AT VI, pp. 19-20).

Although mathematics is concerned with magnitudes in general, if the intellect is to be aided by imagination, we must inevitably deal with particular magnitudes, as those are the ones we encounter in our imagination. On that note, Descartes emphasizes that whatever is true of magnitudes in general is also true of any magnitudes in particular. However, it is most useful to apply what holds of magnitudes in general to the "species of magnitude which gets depicted in our imagination most easily and distinctly of all" (AT X, p. 441). This is the "real extension of a body, abstracted from everything else other than that it is shaped" (Ibid.) since "in no other subject are all the differences of the proportions exhibited more distinctly" (Ibid.).

Thus, the depiction of magnitudes in the imagination in the form of shapes or figures of bodies must be done in a way that enhances distinctness. As mentioned earlier, this point is also made in the *Discourse on the Method*. Just as later in the *Principles of Philosophy*, Descartes here understands by 'extension' everything that has length, breadth, and depth. He emphasizes that, as far as the purview of the imagination is concerned, extension is not something different and separate from its subject. Extension, when conceived abstractly or as something truly distinct from that which is extended, is a product of the intellect alone and cannot be grasped by the imagination as such (AT X, p. 444).

The same applies to concepts like 'extension', 'shape', 'number', 'surface', 'line', 'point', 'unity', etc. In fact, propositions such as "Extension or shape is not body", "A number is not the thing numbered", "A surface is the limit of a body", "A line is the limit of a surface", "A point is the limit of a line", and "Unity is not a quantity" are only true if abstracted from the imagination. When not so abstracted, however, we imagine a subject conceivable in terms of a multitude, set, or collection of (extended) units whenever we think of a number. Similarly, when dealing with figures or magnitudes we are dealing with a body or extended subject conceived as having nothing but its shape and, as such, it must have length, breadth, and depth.

However, if we conceive a given body as a surface, we are leaving out—though not denying, but abstracting from or putting into brackets—its depth, when conceiving it as a line, we are leaving out both its depth and breadth. When conceiving it as a point we are omitting "every other property save its being an entity (*omisso omni alio, praeterquam quod sit ens*)" (AT X, p. 446). Finally, when conceiving it as a solid we are considering all these dimensions. As a matter of fact, Descartes goes so far as to imply that extension and quantity are not really different and qualifies such distinction as a subtlety—a thesis fully stated later in his *Principles of Philosophy*, as mentioned earlier.

In line with his earlier goal of reducing proportions to "a clear equality between what is sought and something which is known", Descartes asserts that it is sufficient to focus only on the features of extension that help elucidate differences in proportion: namely, dimension, unity, and figure. 'Dimension', in Descartes' sense, refers to a mode or aspect by which a subject is considered measurable. This includes our familiar mathematical dimensions such as length, breadth, and depth (which make up extension), but also indefinitely many others (AT X, pp. 447-8). Unity is the common nature (that is, common to both body and mind) which all the things being compared must equally participate in. Lastly, as noted earlier, figure refers to an extended subject or body considered as stripped away of everything but its shape.

In representing relations and proportions through figures, we compare either multitudes or magnitudes. The former corresponds to discrete quantity, number, and order and the latter to continuous quantity, magnitude, and measure. In some cases, magnitudes can be converted into multitudes[19]. However, a relation of order can be instantiated by only two related elements, whereas a measure always requires a third element as a common unit of measure. In order to represent multitudes by means of figures, Descartes uses sets of points (including figurate numbers), and for magnitudes, he uses rectangular surfaces or straight lines (he also considers lines and points figures). In this way, we obtain diagrammatic representations of both kinds of quantity.

Descartes is explicit in stating that it is helpful to "draw these figures" and "display them before our external senses" (AT X, p. 453), as this 'visual display' aids us in forming more distinct images in our imagination. Figures also play a crucial role in maintaining *attention* in our thought (*cogitatio*), which, as we noted earlier, is a mark of intuition[20]. It is also important to highlight that in the *Rules*, Descartes limits himself to one- or two-dimensional figures for representing magnitudes. There are no diagrams of solids. This is, however, no accident:

> The final point we should bear in mind is that among the dimensions of a continuous magnitude none is more distinctly conceived than length and breadth, and if we are to

19 In some cases, we might view a given magnitude as composed of a sum of units of measures, such as five meters, 'five' being here a natural number counting units of measure (meters) and not the corresponding magnitude. This does not work, however, when the ratio between the unit of measure and the magnitude measured does not correspond to a whole number.

20 Descartes is emphatic in distinguishing between intellectual intuition (*mentis intuitus*) and visual intuition (*intuitus oculorum*), AT X, p. 368; 400-1.

> compare two different things with each other, we should not attend at the same time to more than these two dimensions in any given figure. For when we have more than two different things to compare, our art demands that we survey them one by one (*succesive percurrere*) and attend to no more than two of them at once (AT X, p. 452).

In a one- or two-dimensional figure, everything is, so to speak, 'open to view' at once. Thus, the use of no more than two dimensions serves the purpose of distinctness, since "among the dimensions of a continuous magnitude, none is more distinctly conceived than length and breadth." Descartes' restrictions therefore aim to more effectively aid the intellect through diagrammatic representations in our imagination. Having addressed the use of figures in *Rules XIV* and *XV*, let us now turn to the discussion of symbols in *Rule XVI*.

Symbols (*notae*, *chiffres*) are introduced as abbreviations of figures. As a consequence, when these abbreviations are the 'briefest possible' or 'very concise', they prevent the imagination from being overwhelmed by the profusion of saturated figures and eliminate unnecessary information, thus freeing the attention of the mind. The information encoded in the symbols can also be written down, allowing it to be 'stored' when it does not "require immediate attention of the mind" (AT X, p. 454). In this way, symbols also serve a mnemonic function, as their written form alleviates the burden of memory by summarizing the information they represent.

Thus, they make it "impossible for our memory to go wrong" (AT X, p. 454), which is especially beneficial for deduction, given its successive nature. In this way, symbols also facilitate the 'continuous movement' of thought in deductions, effectively bridging the gaps, so to speak, in our step-by-step inferences:

> Because memory is often shaky (*labilis*) and so that we might not be forced to spend some part of our attention on refreshing it while we are engaged with other thoughts, art has most conveniently invented the practice of writing. Relying on this as an aid, we shall here commit nothing at all to memory, rather, relinquishing the fantasy freely and completely to the ideas present to it, we shall put down on paper whatever things have to be retained (*quaecunque erunt retinenda in charta pingemus*). And we shall do this by means of very concise symbols (*brevissimas notas*), in order that, after we have distinctly inspected individual items. And we shall do this by means of very concise symbols, in order that, after we have distinctly inspected individual items, in accordance with the ninth rule, we may be able in accordance with the eleventh rule, run through them all with the swiftest movement of thought and to intuit simultaneously as many as possible (AT X, pp. 454-5).

However, symbols are not only abbreviations of figures but also of words, phrases, and sentences in ordinary language. A concise symbol such as $2a^3$, for instance, captures all the information contained in the profusive ordinary expression 'twice the magnitude symbolized by the letter a' while eliminating any superfluous elements:

> Thus, if I write '$2a^3$' then that will be the same as if I were to say 'twice the magnitude symbolized (*notatae*) by the letter a, which magnitude contains three relations (*relationes*)'. By means of this device (*industria*), we shall not only be saving a lot of words (*multorum verborum compendium faciemus*), but—which is the important point—we shall also be displaying the terms of the difficulty so purely and so barely that, although nothing useful would be omitted from them, nothing superfluous is ever found in them either—nothing which might needlessly occupy the capacity of the *ingenium* while the mind is having to encompass many things at once (*dum simul erunt mente complectenda*) (AT X, p. 455).

Having thus exhibited the role of diagrams and symbols in mathematical invention according to Descartes through an analysis of *Rules XIV-XV* and the *Discourse on the Method*, I believe it will be useful, in the next section, to compare his views on this matter with those of Hobbes, in order to clarify their main points of agreement as well as their primary areas of divergence.

V - Descartes and Hobbes on the role of symbols in mathematics

In contrast to Descartes, Hobbes was not a supporter of modern symbolic algebra. On the contrary, he was a vocal critic of modern mathematical methods. While he acknowledged signs as playing the central role in thinking and reasoning, especially in science, as we will see, Hobbes, unlike philosophers such as Descartes and Bacon, was more of an enthusiast of the use of words and ordinary language than any symbolic devices. Nevertheless, like Descartes, he viewed what he referred to as 'marks' (*notae*) primarily as having a mnemonic function. Marks are "sensible things taken at pleasure, that, by the sense of them, such thoughts may be recalled to our mind as are like those thoughts for which we took them" (EW I, pp. 13-4).

On the other hand, what Hobbes called arbitrary signs (signs in general being the "antecedents of their consequents, and the consequents of their antecedents", and they can be divided in natural signs such as smoke is a sign of fire, and arbitrary signs, made by convention) are not used for remembering or thinking but for communication (EW I, p. 14). When words (or *voces* in Latin) are employed as signs of our thoughts, they form what Hobbes calls speech (in Latin, *oratio*), and parts of speech are called names (EW 1, p. 15). Names can function both as marks and as signs. A proposition, in contrast, is a combination of names, and it is primarily with them that our reasoning—what

Hobbes considers a kind of computation or calculation—is concerned (EW III, p. 23; AT VII, p. 178).

Despite conceding such an important role to marks and signs in science, Hobbes was a harsh critic of the use of symbols in mathematics, particularly in the context of his censure of mathematician John Wallis. His criticism, however, extends beyond this specific case. In his *Examinatio et Emendatio Mathematicae Hodiernae*, Hobbes asserts, mentioning the symbolic methods of Viète, Oughtred, Harriot, and Descartes, that "whatever could be discovered through these new symbols (*symbola*) could just as well have been discovered through the oldest symbols (*symbola*), namely, words" (OL IV, p. 7). He even argues that, since Diophantus had already used symbols in his *Arithmetic*, Viète "added nothing to Algebra, only replacing the old marks (*notas*) with letters of the alphabet" (OL IV, pp. 8-9).

For Hobbes, as for Descartes, symbols are abbreviations. However, they differ drastically in their view on the usefulness of symbols. While Descartes believes that using the briefest symbols enhances the distinctness of our ideas by allowing us to visualize relations, easing the burden of the mind, and preventing profusion in the imagination, Hobbes, in contrast, argues that symbols only introduce an additional layer of difficulty, complicating our understanding of the subject matter:

> Is it not harder to memorize the force of demonstrations written in symbols (*symbolice scriptarum*) than in Latin? And although Analysis may be written more briefly with symbols (*symbola*) than with full sentences (*oratione plena*), it is not clearer, nor is it accessible to as many people. This is partly because the same symbols are common to few (*eadem symbola paucis sint communia*), and partly also because the words themselves, not just the symbols, must still be run through by the mind at once (*animo simul percurrenda sunt*) (OL IV, p. 8).

This, of course, concerns the use of symbolism in proofs, i.e., in the context of communicating a result. This is perhaps Hobbes' most recurring point on the use of symbols in mathematics:

> For though you understand something by it, you demonstrate nothing to any Body, but those who understand your Symbolique tongue, which is a very narrow Language. If you had demonstrated it in Irish, or Welsh, though I had not read it, yet I should not have blamed you, because you had written to a considerable Number of mankind, which now you do not (EW VII, p. 264).

Hobbes' argument is that the reader would ultimately need to translate the symbols back into ordinary language anyway to make them intelligible and grasp their meaning. Therefore, encoding words in symbols from the outset would be useless and only makes understanding the proof more laborious:

> I shall also add, that Symboles though they shorten the writing, yet they do not make the Reader understand it sooner then if it were written in words. For the conception of the Lines and Figures (without which a man learneth nothing) must proceed from words, either spoken or thought upon. So that there is a double labour of the mind, one to reduce your Symboles to words (which are also Symboles) another to attend to the Ideas which they signifie (EW VII, p. 329)

Of course, these objections presuppose that one is not already familiar with the notation in question or fluent in its use, which would eliminate the need to translate it back to one's own language. Nonetheless, Hobbes seems to concede a role for symbolism in mathematical invention, or at least in formulating a proof: "Symboles are poor unhandsome (though necessary) scaffolds of Demonstration; and ought no more to appear in publique, then the most deformed necessary business which you do in your Chambers" (EW VII, p. 248). In Hobbes' metaphor, If symbols are like scaffolds—meant to be removed once the structure is complete—then they clearly perform a major role in the invention of the result, much like scaffolds do in the construction of a building.

This suggests that Hobbes approves the use of symbols as marks, but not as signs, to use his own terminology. In contrast, Descartes not only acknowledges the importance of symbolism in mathematical invention but also has no reservations about using symbols when presenting proofs—as evidenced by his own mathematical work. One could even argue that, by facilitating the "continuous movement of thought," symbols can even *improve* the understanding of a proof. Yet, Hobbes thinks that, even when used successfully, "Symboles serve only to make men go faster about, as greater Winde to a Winde-mill" (EW VII, p. 188). This may sound like a positive thing, but Hobbes surely does not see it as a reason to give symbols any special praise. Furthermore, although he acknowledges the need for symbols in invention, he seems to assign them a relatively minor role:

> But this doth not properly belong to algebra, or the analytics specious, symbolical, or cossick ; which are, as I may say, the brachygraphy of the analytics, and an art neither of teaching nor learning geometry, but of *registering with brevity and celerity the inventions of geometricians.* For though it be easy to discourse: by symbols of very remote propositions; *yet whether such discourse deserve to be thought very profitable, when it is made without any ideas of the things themselves, I know not* (emphasis added, EW I, pp. 316-7)

As we saw earlier, for Hobbes, symbols mostly serve a brachigraphic function—they act as abbreviations or shorthands for full expressions in ordinary language, with the primary purpose of "registering with brevity and celerity the inventions of geometricians." This passage also shows how Hobbes', like Descartes, is concerned that we use symbols attending to their meanings. Additionally, Hobbes is also a major critic of profusion in notation, as evidenced by his criticisms of Wallis. He compares Wallis' notation to both a scab of symbols (EW VII, p. 316) and the traces left by a hen scrapping (EW VII, p. 330).

To summarize, Hobbes and Descartes both have a primarily abbreviatory conception of symbols, viewing them primarily as aids for memory. Both philosophers also criticize notational profusion. However, Hobbes does not approve of using symbols to communicate mathematical proofs; he believes proofs should be rendered in ordinary language. Descartes, on the other hand, appears to have no problem with the use of symbols in proofs. Moreover, Descartes argues that the briefest symbols can provide distinctness to our ideas, whereas he sees ordinary language as a source of error, obscurity, and prejudice. Symbols, according to Descartes, help prevent the profusion of figures and words from overwhelming our imagination. In contrast, Hobbes asserts that symbols cannot accomplish anything that words cannot do, albeit more slowly.

VI - Concluding remarks

Descartes was not an advocate of the idea that language and signs of any kind are constitutive of our thought, as Hobbes and Leibniz believed. However, as we hope to have shown, he did believe that some use of signs—such as diagrams and symbols—plays a prominent, though not essential, role in mathematical invention. While we do not require images to have knowledge of extension (or quantity) and ideas of extended objects, existing or not, imagination significantly improves our knowledge of extension and its modes. Mathematics, according to Descartes, is the science that is the most helped by imagination. In this paper, I have argued based especially on *Rules XIV-XVI* and the *Discourse on the Method*, that this assistance is largely provided by figures and symbols.

Figures depict quantities, their relations and proportions, most easily and distinctly, and enhance attention. Concise symbols (*notae*, *chiffres*), on the other hand, prevent profusion from overwhelming our imagination and assist memory, particularly in deductions. I have also argued that the way in which imagination can be said to make our knowledge of extension and its modes better is mainly by providing our ideas with distinctness, and that both diagrams and symbols are important in this process—in contrast to ordinary language, which is a source of error, prejudice, and obscurity. Finally, I believe there is little evidence to suggest that Descartes' views on the nature and role of diagrams and symbols have changed significantly since his early methodological works, just as his views on the role of imagination in mathematics appear to have remained largely unchanged.

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