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Non-octave-repeating scales as a form of tonal organization

Escalas sem repetição de oitava como uma forma de organização tonal

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ABSTRACT: Whilst most of the world's scales are octave-repeating, the fact is that non-octave-repeating scales have been employed for organizing tonal content in different musical cultures from the Middle Ages to present day. However, there have been few academic studies that deal with the subject from a global perspective, beyond the application of certain non-octave-repeating scales in specific musical contexts. Hence, the extent to which these structures have served for determining pitch content in music has until now not been widely extended or readily available to researchers and musicians potentially interested in the issue. The present paper draws from a variety of different sources in order to present a detailed and structured overview of the use of non-octave-repeating scales in a wide range of musical settings, thus confirming the hypothesis that these structures have proven to be a useful and time-tested way of organizing pitch content in music.
KEYWORDS: Non-octave-repeating scales; Music theory; Musical analysis; Scale theory; Composition; Improvisation.

RESUMO: Embora a maioria das escalas do mundo seja repetida na oitava, o fato é que escalas sem repetição de oitava têm sido empregadas para organizar o conteúdo tonal em diferentes culturas musicais desde a Idade Média até os dias atuais. No entanto, há poucos estudos acadêmicos que tratam do assunto de uma perspectiva global, além da aplicação de determinadas escalas sem repetição de oitavas em contextos musicais específicos. Portanto, a extensão em que essas estruturas serviram para determinar o conteúdo da altura na música não foi até agora amplamente ampliada ou prontamente disponível para pesquisadores e músicos potencialmente interessados no assunto. O presente artigo se baseia em uma variedade de fontes diferentes para apresentar uma visão geral detalhada e estruturada do uso de escalas sem repetição de oitava em uma ampla gama de ambientes musicais, confirmando, assim, a hipótese de que essas estruturas provaram ser uma forma útil e comprovada de organizar o conteúdo de altura na música.
PALAVRAS-CHAVE: Escalas sem repetição de oitava; Teoria da música; Análise musical; Teoria das escalas; Composição; Improvisação.

1. Non-octave-repeating scales as a form of tonal organization

The vast majority of music draws its tonal content from musical scales. Consequently, musical scales and the different ways in which these are applied are quintessential in defining the sonority of music across the globe. As can be expected by how wonderfully diverse music sounds in different cultures and traditions, a wide range of scales exist, used by human beings to make music. These differ in multiple aspects, from pitch content and intervallic structure to the actual range of available pitches from which the scales themselves are drawn. Yet one commonality is so extended that it is practically a universal, being even regarded as a defining trait in most texts: musical scales repeat at the octave. In other words, non-octave-repeating scales are not generally considered a practical scalar option.

Hence, scales that repeat at the octave are found in all musically developed cultures (Burns 1999). Even in those where the interval of a perfect octave is not present, as in Javanese Gamalean music, a slightly sharp or flat pseudo-octave takes its place as the point of a scale's conclusion and repetition in the next octave (Sethares 2005). This is quite logical, given that human beings perceive pitches at the octave to be equivalent, a characteristic referred to as tone chroma or octave-equivalence (S. Brown and Jordania 2013). When two pitches sound at the interval of a perfect octave, the effect is comparable to that of a perfect unison, which is evidenced whenever men, women and children sing the same melody together (Trehub, Becker and Morley 2015). So, since we identify pitches in different octaves as equivalent, it is logical that the scales from which melodies are drawn should repeat at the octave. Given how we perceive pitch relations, the contrary would seemingly be unintuitive.

Furthermore, there are structural considerations that argue against non-octave-repeating scales as a suitable form of tonal organization. As noted by Tymoczko, any pitch-class set, while unique to itself as an octave-repeating structure, could potentially belong to an infinite number of non-octave-repeating scales (Tymoczko 2004). The C diatonic scale, for instance, is the unique octave-repeating scale containing only the pitch classes 0, 2, 4, 5, 7, 9 and 11; yet these pitches could be arranged in any number of ways in non-octave-repeating scales. This, naturally, applies to all scales and pitch sets. As a result, the infinite theoretically possible arrangements of any given pitch set in non-octave-repeating scales make them unlikely candidates to be favored by human beings for making music. Also, given the potential range of such scales, these could easily contain the full pitch class set. In the light of what we know about octave-equivalence, such a situation would foreseeably simply lead to chromatic music, as the scale would contain the complete aggregate and we perceive pitches to be equivalent, regardless of which octave they are sounded in.

If, despite a seemingly solid case for why non-octave-repeating scales are simply not practical for organizing pitch content in music, these indeed had been extensively employed in different musical cultures over a considerable period of time, one would expect the subject to have been deemed a phenomenon worthy of study, documenting and noting. If non-octave-repeating scales had in fact been favored and successfully applied by musicians in a variety of different contexts, one certainly would not expect the use of such structures to pass unnoticed.

Yet, surprisingly, although many musicians and researchers remain unfamiliar with the topic, non-octave-repeating scales have been used as a form of tonal organization from Byzantine times to the present day, even preceding functional harmony and major/minor tonality by several centuries. The fact

is that one of the earliest known musical treatises includes a non-octave-repeating scale and there are composers and improvisors today that continue to explore the possibilities inherent in such structures.

However, to the best of our knowledge, a study focusing expressly on the application of non-octave-repeating scales as a form of tonal organization is currently missing in the existent literature. The few academic texts that do deal with the subject, generally do so within a wider discussion of a specific musical situation or culture (Swan 1940a). Thus, the subject has generally passed unnoticed by the wider academic community.

This paper seeks to fill this gap in the existing literature by confirming the general hypotheses that non-octave-repeating scales are a viable and time-tested form of tonal organization by presenting an account of the use of these in different musical cultures from the Middle Ages to present day. The issue has been partially dealt with in Crowley and Gómez (2023), yet not at length. Whilst a brief account of the application of non-octave-repeating scales in musical practice is provided in said article–the main purpose of which is to isolate a previously undefined group of two-octave scales based on common structural properties–, the present paper discusses the issue in greater depth, so as to present a thorough overview and analysis of the application of non-octave-repeating scales throughout the history of musical practice.

In doing so, we hope to bring this topic to the attention of the wider academic community and that it will therefore serve as a point of departure for further discussion and research into the topic.

2. Use of non-octave-repeating scales before the twentieth century

Although one might expect a notion such as non-octave-repeating scales to have developed in the avant-garde of the twentieth century, the origin of such structures predates European major-minor tonality by numerous centuries.

2.1. The Enchiriadis scale

The anonymous ninth century treatise Musica Enchiriadis contains a non-octave-repeating scale with a two and a half octave span. The eighteen-pitch scale is known as the Enchiriadis Scale or the Daseian Scale, given the Daseian notation used throughout the treatise (Erickson 2001) and consists of a series of four disjunct T-S-T tetrachords repeating at the fifth plus an additional incomplete tetrachord of only two pitches at the top (see Figure 1).

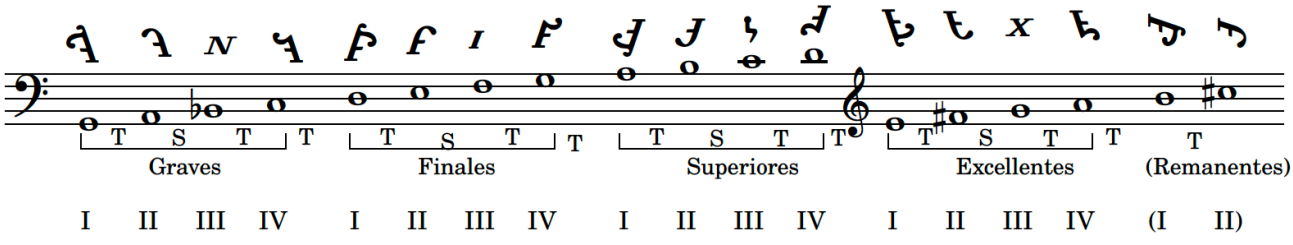

Figure 1 – Enchiriadis scale in modern staff and Daseian notation
Reference: Own elaboration after Cohen (2008)

The scale is most likely of Byzantine origin and the notes within it are classified according to their position within the tetrachord and named by Greek ordinal numbers, which also denote the four basic modal categories in Byzantine tradition: I=*protus*, II=*deuterus*, III=*tritus*, IV=*tetrardus* (Cohen 2008). Since the intervallic structure is repeated at the fifth, each of the four notes within the tetrachord can act as the tonic of a distinct "fifth-repeating" mode. Thus, the names of the individual pitches refer to both the pitch itself and the modal quality it embodies, given its place within the tetrachord. As Cohen explains,

> The first note of each and every tetrachord has the same modal quality (*protus*), and thus the same name (*protus*), and so too for the other three notes. That is, since there are only four different tetrachordal positions, each of which creates (by its pattern) one of the four basic modal qualities by which the notes are identified, there are, in a sense, really only four truly different' notes, all the others being replications of these at higher and lower levels of pitch, separated by perfect fifths. (In a similar way, the octave periodicity of our modern system makes it possible to operate with pitch classes.)(Cohen 2008, 325–326)

The concept of modal quality is integral to pitch conception in the Enchiriadis scale and the scale has a clear pedagogical and theoretical purpose, namely, to explain and help the student distinguish the four "modal qualities" associated with each note—*protus*, *deuterus*, *tritus* and *tetrardus* (Cohen 2008, 325–326). Each tetrachord is repeated at the interval of a perfect fifth, creating a periodicity analogous to that found in octave-repeating scales. In fact, as can be observed in many of the examples throughout the text, most non-octave-repeating scales tend to repeat at an interval smaller or even greater than the octave. In the case of the Enchiriadis scale, the notes which constitute the repeating tetrachord are indeed treated as equivalent throughout the scale, creating a periodicity and general pitch space much like that of octave-repeating-scales, such as the diatonic scale in Western Music. Although it is clear that the Enchiriadis scale fulfilled the pedagogical and theoretical purpose of explaining the modal qualities of melodies (Cohen 2008, 326), it has also been suggested that it represents the actual tone system used in plainsong (Maloy 2009, 75).

2.2. The Znamenny scale and other fourth- and fifth-equivalent pitch sets

Scales based on a succession of similar trichords/tetrachords were employed in Medieval Western Europe—and in countries such as Russia, Bulgaria, Georgia, Azerbaijan or Armenia—as a melodic framework in Christian plainchant, ultimately influencing the traditional music of various Eastern Orthodox regions and cultures (Nikolsky 2016, 8–9). These scales are constructed by stringing together similar trichords/tetrachords, generally at the interval of a fourth or fifth, throughout various octaves. Hence, pitch content differs in consecutive octaves. The resulting scale is treated as a single entity and is especially pronounced in a polyphonic setting (Nikolsky 2016, 8). An example of such a scale is that of the Znamenny rospev, the chant of the Russian Orthodox church. Most likely a transformation of Byzantine chant which entered Russia in the tenth century at the time of Christanization (Swan 1940b, 232), its name, literally "chanting by signs", is a reference to the neume system originally used to notate the music (Velimirovic *et al.* 2001). The system—initially very similar to Early Byzantine Notation (Swan 1940b, 232), indicating only intervallic relations rather than discrete pitches (Velimirovic *et al.* 2001)—gradually evolved into a form of pitch-specific notation. The gamut encompassing the notes of the melodies used in the chants most likely emerged in the first major notational reform of the seventeenth century, associated with the theorist Ivan Shaydur (Velimirovic *et a*l. 2001). It consists of four consecutive major trichords at the distance of a semitone, forming a scale with an intervallic span of a perfect twelfth.

The scale is fourth-equivalent, as opposed to octave-equivalent, resulting in a non-octave-repeating scale structure. As shown in Figure 2, the scale contains the pitch B in the first octave and Bb in the second. If the register were to be extended through the addition of further trichords, more differential pitches would appear in successive octaves.[1] Yet, since Znamenny chants are strictly a cappella and until the seventeenth century Russian liturgical music was purely monophonic (Buelow 2016, 444), the scale's intervallic span is limited to the vocal range employed in the chants. The trichords delimit the four different registers or accordances of the scale and are named from the lowest upwards "Low", "Sombre", "Bright" and "Triple Bright" (Wall 2009, 23), as shown in Figure 2. Traditionally, singers would commence a melody in the lower register of the scale and progress in stepwise fashion, finally reaching the triple-bright accordance (Swan 1940a, 367).

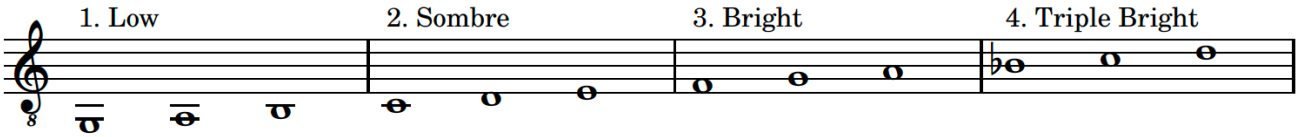

Figure 2 – Znamenny chant registers
Reference: Own elaboration after Wall (2009)

Melodic motion within Znamenny chant music is mostly conjunct, with very few intervallic leaps, mainly of a fourth or fifth at cadential points (Wall 2009, 23). The scale renders three different modes, depending on which note functions as the tonic; taking G as the starting note, the three different tetrachords from which the different modes could be constructed would be G major, A minor and B diminished or Phrygian (Nikolsky 2016, 9). The Znamenny scale is also prominent in Russian folk music, affecting even the construction of certain instruments, such as the Livenka accordion, the melody side of which consists of said pitch set (Belianski 2015, 21).

Similar fourth-equivalent scales can be found in other musical cultures. The basic scale in Armenian music, for instance, also consists of a chain of conjunct major tetrachords, meaning that the highest note of one serves as the starting point for the next (See Figure 3). Being intervallically similar to the Znamenny scale, the Armenian scale also renders three different modes. Since the "relations between the degrees are repeated after every fourth note, three types of mode can be formed naturally, corresponding to the G, A and B modes; these constituted the original basis of Armenian music" (Pahlevanian, Kerovpyan, and Sarkisyan 2001). The melodic motion in Armenian music is also mostly conjunct. "Skips (primarily upwards) occur, if at all, in opening structures, which are always completed with a wave-like movement" (Pahlevanian, Kerovpyan, and Sarkisyan 2001).

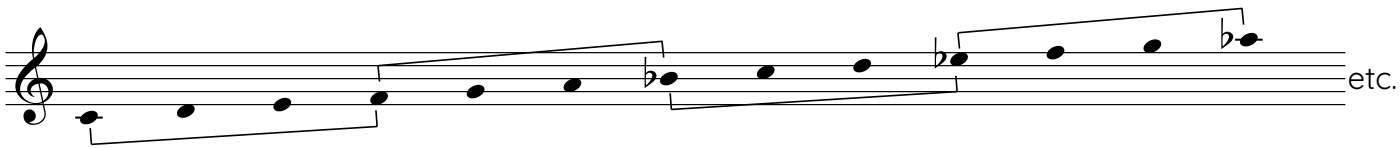

Figure 3 – Basic scale in Armenian music
Reference: Own elaboration after Pahlevanian, Kerovpyan, and Sarkisyan (2001)

[1] The idea of expanding the scale in this manner was exploited by the Russian composer Yuri Butsko and will be discussed in Section 4.4.

One of the main scales used in Jewish liturgical chanting, known as the Adonai Malakh, is also based on a similar succession of tetrachords (See Figure 4). This mode, or *shteyger*, is named after Psalm 93 that is sung at the end of the Kabbalat Shabbats service: *"Adonay malach geut lavesh, lavesh hashem oz hitazar"* (The LORD reigns, he is robed in majesty; the LORD is robed in majesty and armed with strength).[2] The scale, often blended with the European major scale, is based on a series of conjunct equal tetrachords of 1-1-1/2 tones and is regarded as representing glory and majesty (Seroussi *et al*. 2001).

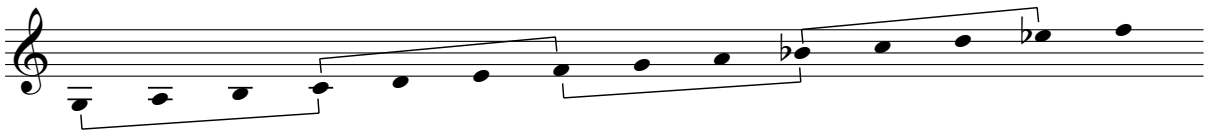

Figure 4. Adonai Malakh scale
Reference: Own elaboration after Seroussi *et al.* (2001)

Whilst the aforementioned music based on non-octave-repeating fourth-equivalent scales is monodic, polyphonic implementations are also common. The differences in pitch content in successive octaves, or "false relations", are much more apparent in a polyphonic setting, where they are more likely to sound in close proximity, due to the vertical organization of the music (Nikolsky 2016, chap. Appendix II). On the other hand, in a monodic context a singer would have to climb an entire octave until the difference in pitch content becomes apparent, a considerable distance considering the stepwise melodic motion of the music based in question. The range of the scale is often extended in a polyphonic context, featuring up to seven trichords in Western Slavic folk songs (Nikolsky 2016, pt. Appendix II).

Nikolsky provides various audio examples of polyphonic music based on such pitch sets. The South Russian Cossak Song (see Musical References) in Figure 5 is based on the scale shown in Figure 6, consisting of a succession of minor tetrachords.[3] The scale contains both C#4 and C5, resulting in an increase of tension as the melody moves into the higher register (Nikolsky 2016, 9).

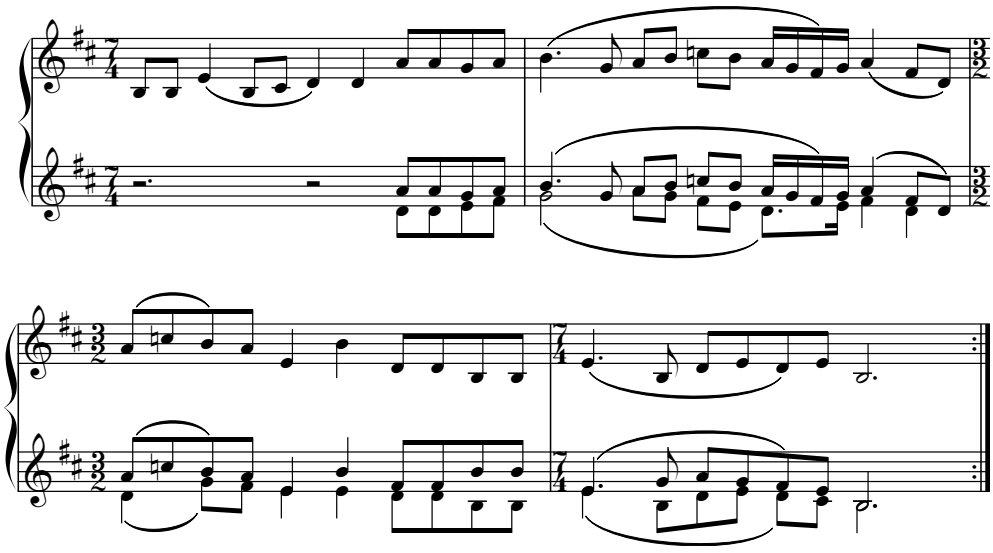

Figure 5 – *Ne oryol li s lebedem kupalisia* - Cossak song
Reference: Own elaboration after Kostenko (1985)

[2] Information retrieved from https://www.jewish-music.huji.ac.il/content/steiger.
[3] The top pitch in Figure 6 (D5) is not actually used in the song but is displayed to illustrate the scale's tetrachordal construction.

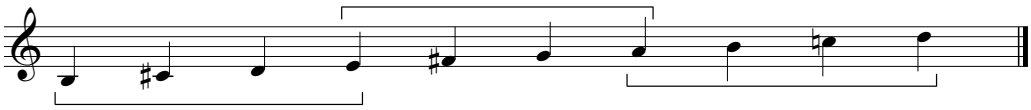

Figure 6 – Scale used in *Ne oryol li s lebedem kupalisia*

The Georgian Lullaby *Kakhuri-Nanina* (see Musical References) also draws its pitch content from such a scale, this time based on the diminished or phrygian tetrachord (see Figure 7). The sensation of increased tension provided by the differential pitch Ab4 in bars 12—13 is especially evident in this piece and effectively increases the emotional intensity of the music.

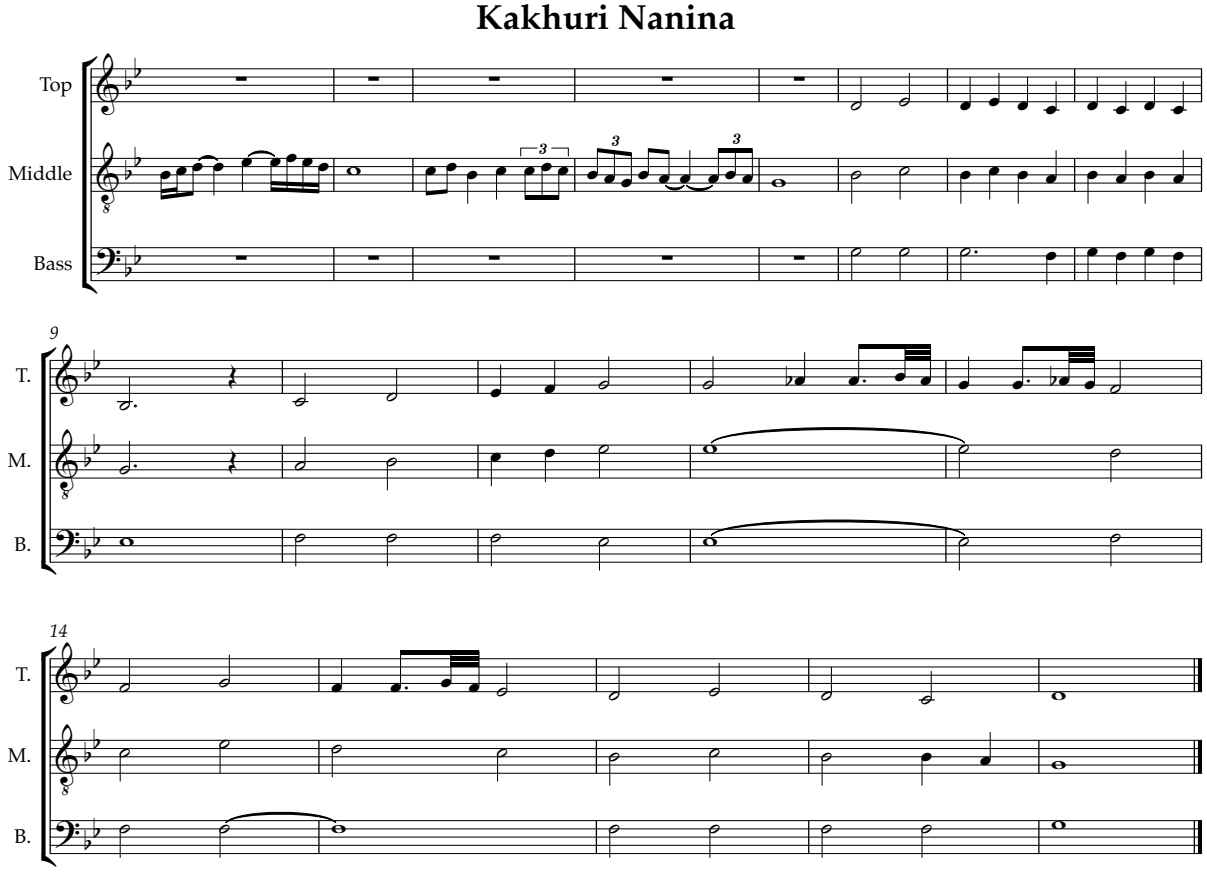

Figure 7 – *Kakhuri Nanina*: Georgian Lullaby
Reference: Own elaboration after Wilkox (2001)

In the same way that a fourth-equivalent scale results from stringing together tetrachords at the distance of a perfect fourth, a fifth-equivalent scale emerges when stringing together pentachords at the distance of a perfect fifth. Such scale structures are found in Georgian traditional vocal polyphony, where the three diatonic scale systems are *a)* fifth-equivalent, *b)* fourth-equivalent and *c)* a combined scale system, based on the former two (Gogotishvili 2010, 218); the three diatonic scale systems are shown in Figure 8. The fifth-equivalent scale moves towards sharps as it ascends, whereas the fourth-equivalent moves towards flats, making the fourth-diatonic more consonant, due to the presence of diminished as opposed to augmented intervals; the combined scale system softens the harsh intervallic structure of the fifth diatonic scale (Gogotishvili 2010, 219).

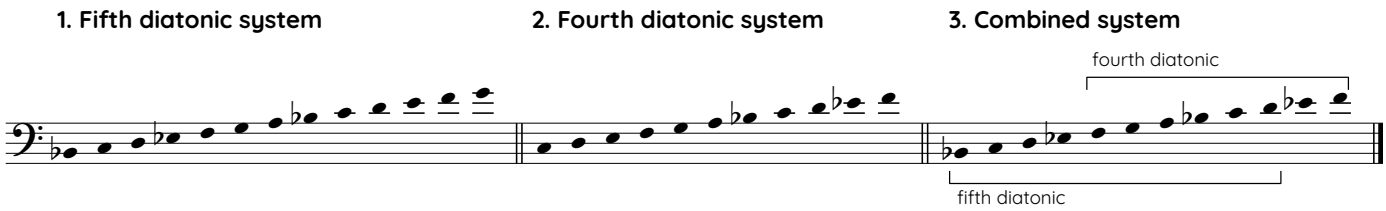

Figure 8 – The scales of the three diatonic systems in Georgian vocal polyphony and their intervallic characteristics
Reference: Own elaboration after Gogotishvili (2010)

2.3. Non-octave-repeating structures in Arabic maqamat

The maqam is the equivalent of a scale or mode in Arabic music. Yet the concept of maqam goes beyond that of a mere scale, providing not only pitch content for tonal organization but, similarly to the Indian raga, further complex features regarding its realization, thus being considered by various ethnomusicologists a phenomenon rather than merely a scale (Yöre 2012, 282). It can be found in array of musical cultures, being the principal melodic concept in Middle Eastern music (Nettl 2007). A schema, more than a scale, it has been described as a "process of melodic movement, and an operational protocol of improvisation according to the models and forms of melodic and rhythmic organization" (Ayari and McAdams 2003, 164), a "set of pitches and of characteristic melodic elements, or motifs, and a traditional pattern of their use" (Nettl 2007).

The jins or ajna ("genus" in English) is the basic melodic unit within the maqam. Consisting of 3—5 notes, it embodies not just a collection of pitches, but also a series of melodic conventions and formulae; "each jins has very recognizable and distinct melodies that have been preserved and transmitted orally for centuries" (Farraj and Shumays 2019, 193). A maqam is constructed by chaining together two, sometimes three jins (Farraj and Shumays 2019, 288). These can be connected in either disjunct, conjunct, or overlapping fashion (Ayari and McAdams 2003, 163).

Whilst most maqam scales are heptatonic and confined to one octave, some show a higher pitch content and a few are not octave-repeating (Farraj and Shumays 2019, 286). "In some maqamat with three ajnas, for instance, the jins at the octave may not duplicate the root jins" (Ayari and McAdams 2003, 163). Such is the case of Maqam Bastanikar (see Figure 9), popular in the Ottoman Turkish repertoire, which "from a scalar point of view, (...) is an extreme example of nonequivalence at the octave" (Farraj and Shumays 2019, 390). Receiving its name from the expression "East Wind" in Arabic, the melancholic Maqam Saba is also non-octave-repeating; as can be seen in Figure 10, in contrast to Maqam Bastanikar, the eighth pitch in the scale is not a repetition of the tonic in the second octave (Farraj and Shumays 2019, 409).

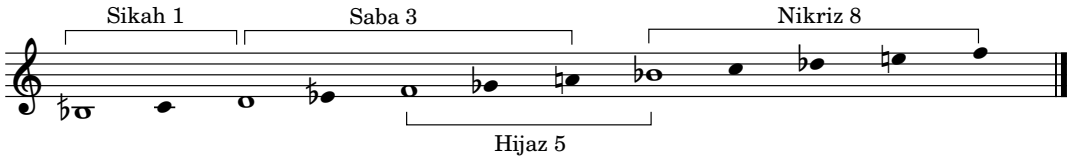

Figure 9 – Maqam Bastanikar. Reference: Farraj and Shumays (2019)

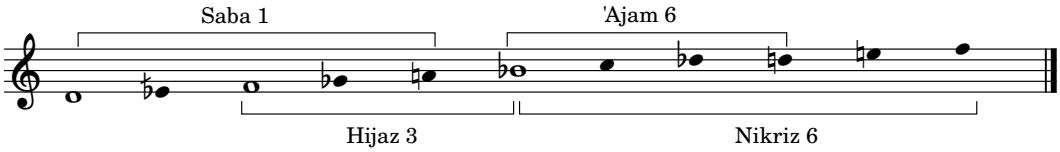

Figure 10 – Maqam Saba. Reference: Own elaboration after Farraj and Shumays (2019)

Due to their construction—by linking different subsets or jins—, non-octave-repeating maqamat such as Maqam Bastanikar are not symmetrical structures, unlike the fourth- and fifth-repeating scales discussed earlier, which, being comprised of a chain of similar subsets, are symmetrical. Arabic music is generally monodic (Farraj and Shumays 2019, 161), meaning the notes differing in successive octaves, as in the case of the examples of monodic application of fourth-repeating scales discussed earlier, will not sound in close proximity to one another like they would in a polyphonic setting.

3. Non-octave-repeating scales in twentieth century Western Musical Practice: Scale construction

Whilst many composers as from the late nineteenth century would abandon the use of scales in favor of a more chromatic approach, others proceeded to experiment with a variety of different scales other than those commonly employed in the tonal system of the Common Practice Period (Tymoczko 2004). Although most scales employed in works pertaining to the present and previous century tend to be octave-repeating, some composers and theorists have devised scales which do not conclude at the octave, spanning a wider intervallic range prior to the repetition of the initial pitch set. Before discussing the application of such scales in composition and improvisation, we will present an overview of different ways in which composers and theorists have devised non-octave-repeating scales as from the twentieth century.

3.1. Multi-octave scale construction in Persichetti

In his seminal textbook *Twentieth Century Harmony*, Persichetti (1961) presents an overview of many of the harmonic procedures used by composers throughout the first half of the twentieth century. In the section dealing with synthetic scales, the author includes structures with a modular span of two or more octaves he refers to as two-octave and multi-octave scales, respectively.[4]

The three methods Persichetti proposes for devising these scales account for most non-octave-repeating scale constructs and yield scales that are structurally very different from one another. Those methods are the following:

- Scale-type a: By a succession of similar tetrachords[5]
- Scale-type b: By a succession of dissimilar tetrachords[6]
- Scale-type c: Through the combination of two different one-octave scales with common tonics

Scales resulting from the combination of two different one-octave scales with a common tonic (scale-type c) will repeat their starting tone in the first octave, which is not necessarily the case with scales resulting from a succession of similar or dissimilar tetrachords (see Figure 11 and 12). Being non-symmetrical scale constructs, as in the case of scales built from dissimilar tetrachords, the number of possible transpositions

[4] Persichetti uses the term two-octave scale to describe a scale spanning said intervallic range and the term multi-octave scale to refer to scales that span three or more octaves. Throughout this text the term multi-octave scale is used to describe non-octave-repeating scales of any octave-range.
[5] The previously discussed fourth- and fifth-repeating scales would fall under this category.
[6] Non-octave-repeating maqamat would fall under this category.

will be equal to the number of semitones contained in the scale's modular interval span, e.g., 24 in the case of a two-octave scale.

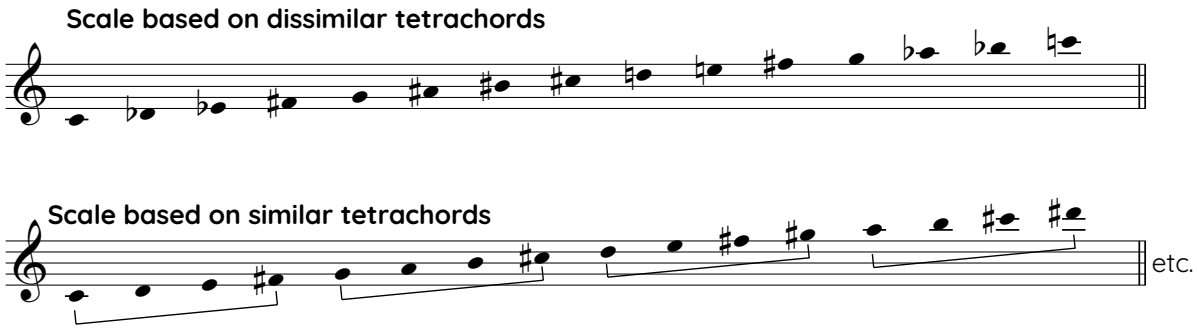

Figure 11 – A two-octave scale built from dissimilar tetrachords and a multi-octave scale built from similar tetrachords
Reference: Own elaboration after Persichetti (1961)

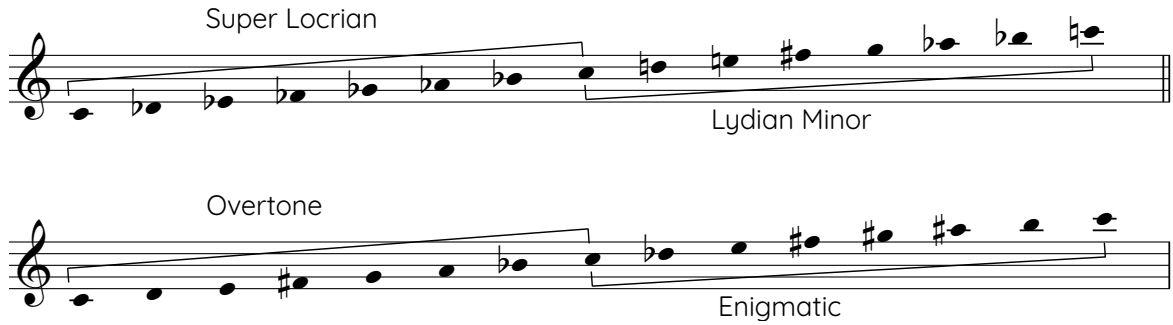

Figure 12 – Scales built by combining two different one-octave scales with common tonics
Reference: Own elaboration after Persichetti (1961)

On the other hand, scales which are the result of a succession of similar tetrachords (scale-type a) will be of a symmetrical nature and will transpose on to themselves after a given number of transpositions, in the same way as Oliver Messiaen's modes of limited transposition do (Messiaen 1944). The number of possible transpositions will be equal to the number of semitones contained within the span of the interval at which the tetrachords are repeated. This can be observed in Figure 13, which shows the different transpositions of Persichetti's example of a multi-octave scale based on the Lydian tetrachord (see Figure 11). Since the tetrachords are repeated at the interval of a perfect fifth, the scale offers seven possible transpositions, after which it transposes back on to the first. In this sense, the generative interval of the scale is a perfect fifth. The concept of a generative interval will be discussed in the following section. Scales of dissimilar tetrachords (scale-type b) may or may not be based on a specific interval cycle and are not symmetrical.

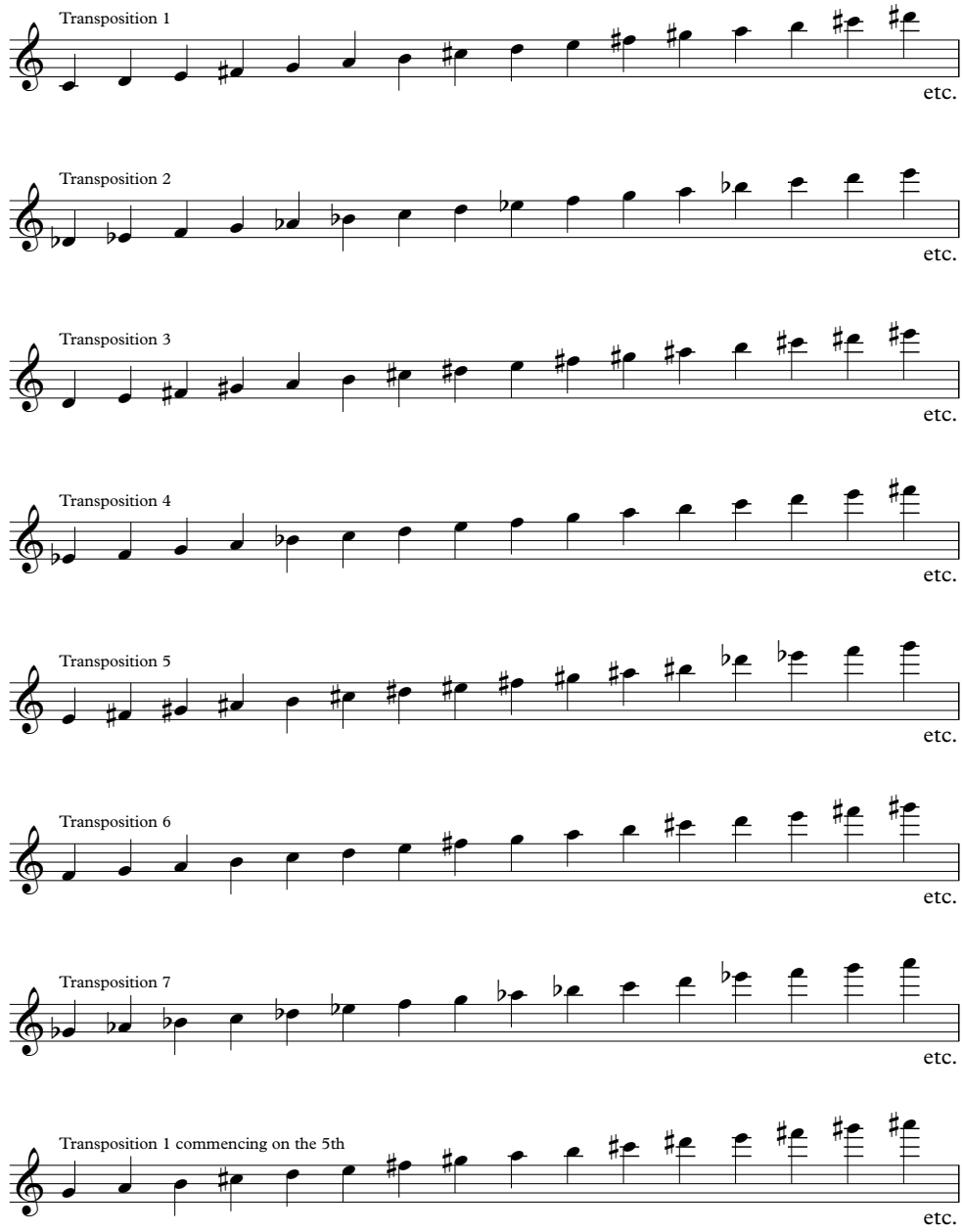

Figure 13 – Possible transpositions of Persichetti's multi-octave scale based on the Lydian tetrachord.

3.2. Scales based on generative interval cycles

Using a generative interval cycle is perhaps the most common way of creating non-octave-repeating scales. In the following section we will discuss various ways in which composers have developed scales in this manner.

3.2.1. Symmetrical non-octave-repeating scales

During the first half of the 20th century, composers and theorists such as Nicolas Slonimsky (1947) or Joseph Schillinger (1946) devised non-octave-repeating scales based on interval cycles that equally divide a given octave-range. The most extensive text published on the subject is Slonimsky's *Thesaurus of Scales and Melodic Patterns*.

First published in 1947, the *Thesaurus* was conceived not merely as a reference book, but as a systematic exploration of unexploited melodic possibilities, intended to provide composers with thematic material they

could draw from in their music. As the author states in the introduction, the publication is "analogous in function with phrase books and dictionaries of idiomatic expression (...), [yet] while phrase books are limited to locutions consecrated by usage, the *Thesaurus* includes a great number of melodically plausible patterns that are new" (Slonimsky 1947, i). The work has been a source of inspiration not only for composers within the Western Art Music tradition, but also to musicians from other genres, such as Frank Zappa (Corcelli 2016, chap. 2) or John Coltrane, whose extensive study of the book during the late mid-late 1950s is considered by many to be one of the principal sources of inspiration for the usage of chord progressions based on major third cycles, found compositions like *Giant Steps* or *Countdown* (Demsy 1991).

The first part of the *Thesaurus* contains 1,033 scales and melodic patterns based on the equal division of one or multiple octaves, commencing with the division of one octave into two equal parts by the interval of a tritone and concluding with the equal division of eleven octaves into twelve equal parts by the interval of a major seventh. Slonimsky arrives at these scales by adding a varying number of notes in similar fashion in between the main pitches that equally divide the total range—a process he refers to as *interpolation*—, rendering symmetrical scales comprised of intervallically identical scale-sections, which will transpose on to themselves after a limited number of transpositions. For instance, a two-octave range may be divided into three equal parts by the interval of a minor sixth (C — G# — E — C). Figure 14 shows two scale options arrived at through the *interpolation* of three notes between these pitches.

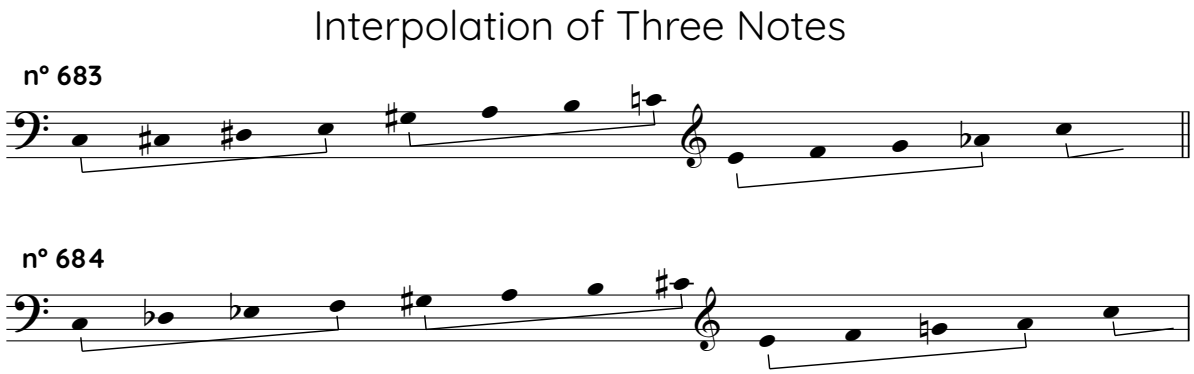

Figure 14 – Scales based on the equal division of two octaves by the interval of a minor sixth
Reference: Own elaboration after Slonimsky (1947)

Similarly, by adding notes above or below the main pitches—labeled *ultrapolation* and *infrapolation* in the text—Slonimsky creates the melodic patterns contained in the *Thesaurus*, which differ from the scales in that they are not linear (see Figure 15).

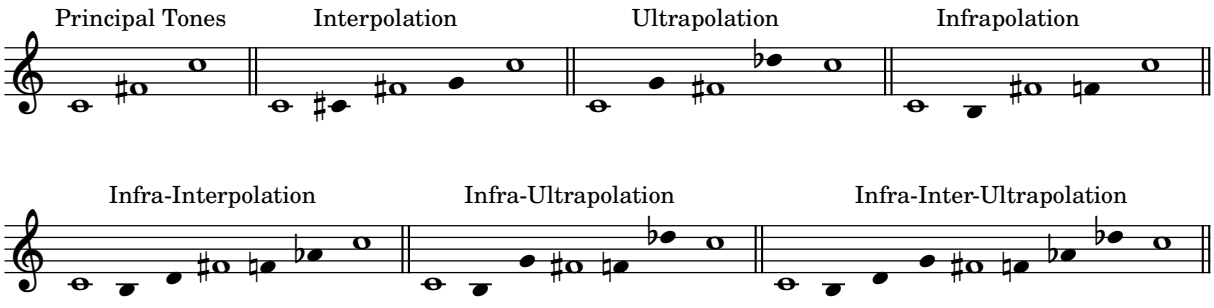

Figure 15 – Inter-, Ultra- and Infrapolation.
Reference: Own elaboration after Slonimsky (1947)

Following this method, Slonimsky presents an extensive collection of symmetrical scales with a modular range of one-, two-, three-, five- and seven-octaves.[7] Constructing nonsymmetrical non-octave-repeating scales based on interval cycles, by stringing together dissimilar scale-fragments at the distance of a regular interval, is far less common, but has been suggested by authors such as Wilcox (1967).

3.2.2. Scales based on interval cycles which span over an octave

The scales found in Slonimsky's *Thesaurus* are based on generative interval cycles arrived at by the equal division of a given octave range. The generative interval defines the length of the scale fragments of which the scale is comprised. Thus, as in Figure 14, a scale based on the equal division of two octaves by the interval of a minor sixth will result in a two-octave scale built of intervallically identical scale fragments spanning a minor sixth. Slonimsky devises scales in this manner by using all intervals within the octave to form the generative interval cycles, except for the semitone, which can only be used to obtain the full chromatic scale.

By following a similar logic, it is also possible to devise non-octave-repeating scales by stringing together scale fragments themselves over an octave in length. Contemporary composers such as Gao Weijie and Joel Hoffman have created such entities to organize pitch content in their compositions. Joel Hoffman's *Piano Concerto*, for instance, is based on a scale consisting of a string of six eleven-note scale fragments with an interval span of a major ninth. Hoffman describes the scale in the program notes to the piece:

> In recent years I've been using a particular kind of harmonic structure for all my music, in which I allow only certain notes to be heard in a given piece. These notes are organized into one very long scale which replicates at some interval other than the octave (as opposed to major and minor scales, all of which replicate at the octave). In "Piano Concert" the interval of replication is the major 9th, the scale has 11 notes in it, and there are 68 notes altogether (in other words, 6 iterations of the 11-note scale + 2 extra notes) (Hofman 2008).

Hoffman's 2006 composition *6-8-2-4-5-8* is also based on a non-octave-repeating scale built of scale fragments which span a major 9th (see Figure 16).[8] Each scale fragment consists of seven intervals, the last pitch of each scale fragment being the starting point of the next. This is repeated a total of six times, equally dividing a seven-octave span by the interval of a major 9th, the pitches of the dividing interval cycle on which the different scale fragments begin being Bb0, C2, D3, E4, F#5 and G#6 (Brown 2019, 43–44).

The Chinese composer Gao Weijie is allegedly responsible for introducing Hoffman to this concept (D. W. Brown 2019, 142). Weijie has used non-octave-repeating scales in compositions such as *The Road* (1996) for violin and piano, in which he employs a non-octave-repeating construct consisting of scale fragments that span 13 semitones, described by Zheng (2017) as follows:

> *The Road* illustrates the technique Gao calls non-octave repeating scale and is one of many approaches Gao takes to pitch cycles. Such scales are inspired by

[7] A section dedicated to scales of a four- and six-octave range is not included, since the dividing interval cycles do not differ from those of a two- and three-octave range, respectively.
[8] An analysis of this piece can be found in D. W. Brown (2019).

Messiaen's modes of limited transposition but are not closed. The non-octave repeating scale alternates tones and semitones, but inserts a consecutive whole tone to prevent closure at the octave and repetition of the cycle. Thus, the series 121212(2)121212(2)12 avoids octave repetition after the seventh tone with the introduction of a consecutive whole step.

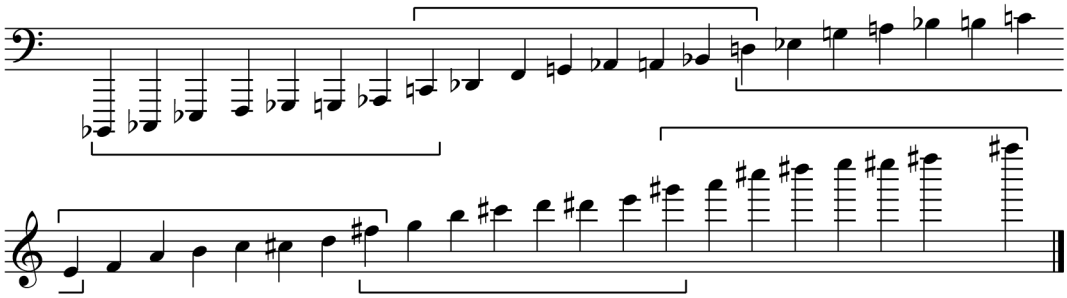

Figure 16 – Hoffman's scale created for 6-8-2-4-5-8 — the brackets show the separate scale fragments.

3.3. Pitch sieves

Non-octave-repeating scales play an important role in the music of the composer and theorist Iannis Xenakis. In order to organize pitch content in many of his works, Xenakis devised large pitch sets spanning various octaves in range. He arrived at these pitch sets or scales through a mathematical process called sieve theory (Xenakis 1992, chap. 7). A sieve is a selection of specific points from the available options along the axis of some dimension (Squibbs 1996, 57). Applied to scale construction, it is a manner of selecting discrete pitches within an intervallic frame through the combination of different interval cycles. More specifically, a sieve is a pair (*M,R*) formed by a *modulus* or *period M* and a *residue R* (or remainder) along with all the elements generated by the formula $R+k\cdot M$, where *k* is an integer. For example, (5,1) gives rise to the infinite set $\{1,6,11,...\} \cup \{-4,-9,-13,...\}$. Logical operations such as union, intersection or complementation are defined on sieves. Xenakis applies the theory of sieves to scales to select pitches according to sieves of different moduli and residues and by performing several of those operations on them. The structures created by Xenakis are based on complex formulas leading to large sets spanning multiple octaves (Squibbs 1996, 60), which do not fall into any of the categories of non-octave-repeating scales listed in Persichetti (1961), discussed in section 3.1. In Figure 17, the sieve used in his piece *Nekuïa* is shown.

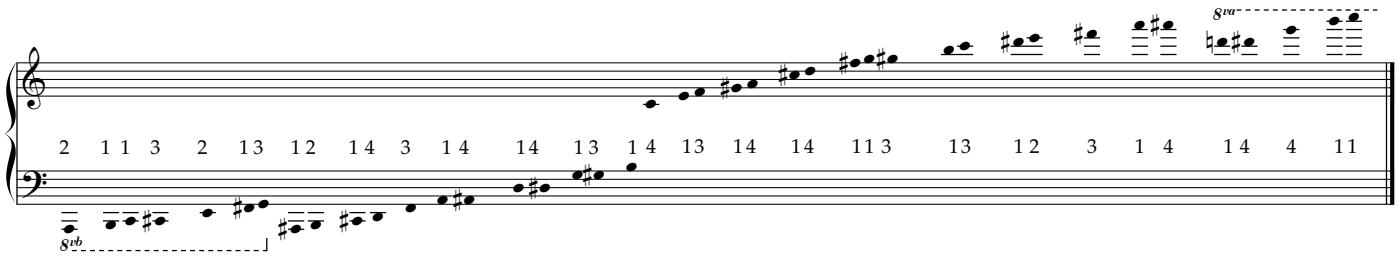

Figure 17 – The sieve of Nekuïa
Reference: Own elaboration after Exarchos and Jones (2011)

4. Application of non-octave-repeating scales in Western Compositional Practice

As is to be expected, non-octave-repeating scales share many characteristics that differ profoundly from those of traditional octave-repeating scales, resulting in a series of unique musical circumstances. The most essential difference is regarding pitch and pitch-class space. In a standard octave-repeating scale, there is no difference between pitch content and pitch-class content, since the scale's pitch set will be repeated in each octave. If the scale contains a C natural, all Cs (C1, C2, etc.) will be present in each repetition of the scale's modular span throughout all octaves. This is naturally not the case with non-octave-repeating scales where pitch content differs in successive octaves. That is, a C natural may be present in one octave, but not in the next; the scale could contain C4 but not C5. Therefore, non-octave-repeating scales are structures which occur in pitch space, as opposed to in pitch-class space (Weston 2020, 8). This is an important distinction that has profound melodic and harmonic implications. Given that pitch content differs in successive octaves, the inversion of a given chord may well result in a different chord altogether, as can a different disposition of the same chord. Scale b) in Figure 18 contains four different forms of a C triad, as can be observed in Figure 19, which displays the different inversions of a C triad within said scale. Figure 20 shows triads in closed and open position built on the root of scale a) from Figure 18; due to the scale's pitch content, the closed position triad is a C minor—comprised of pitches 1-♭3-5—, whilst the open position triad is a C major—comprised of pitches 1-5-10.

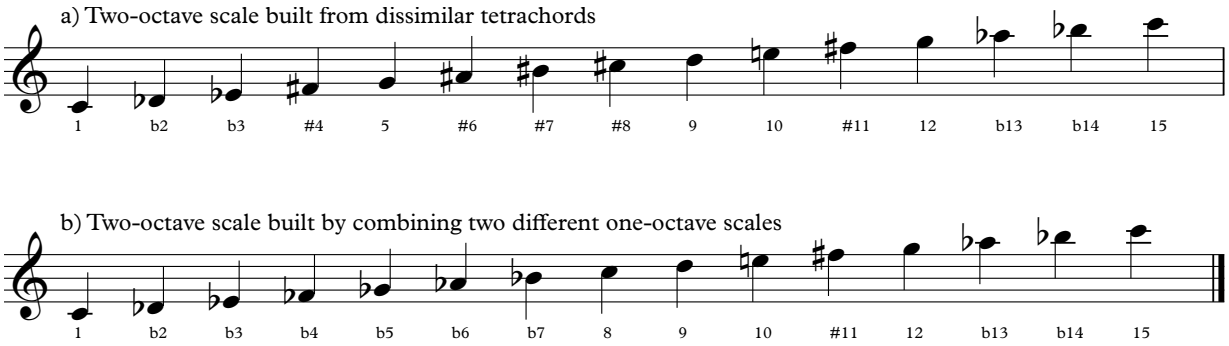

Figure 18 – Pitch content in two scale examples found in Persichetti (1961)

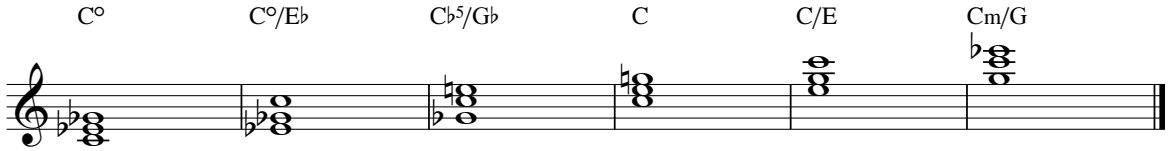

Figure 19 – Inversions of a C triad in scale b) from Figure 18

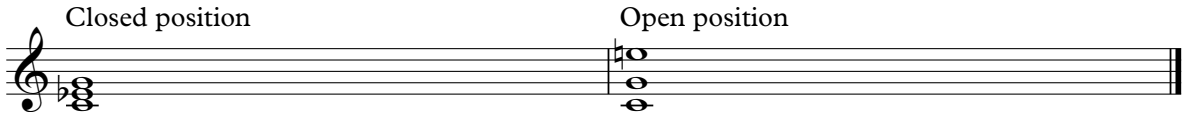

Figure 20 – Closed and open position triads built on the root of scale a) in Figure 18

Although non-octave-repeating scales are far from common in Western Musical Practice, various composers have made use of these structures to organize pitch content in their music. As with many compositional tools and musical concepts developed in the twentieth century, there are no strict rules or guidelines concerning the application of multi-octave scales and the strategies adopted depend largely on the composer and the context of the musical composition. This section seeks to provide an overview of some of those strategies in the music of composers who have employed multi-octave scales in their work in a significative way, though it is perfectly possible that others exist beyond this author's knowledge or are yet to be or are presently being developed.

4.1. Providing structure in an atonal context

Slonimsky suggests three ways of harmonizing the scales and melodic patterns contained in his *Thesaurus*: *a)* by rotation of common major and minor triad inversions not necessarily related to the scale itself, *b)* using master chords—a term he employs to describe dominant 7th chords with the 5th omitted—, or through *c)* autochordal harmonization—a process analogous to traditional diatonic harmonization in major and minor keys, in which "new scales may be harmonized with the aid of chords formed by the notes of the scale itself" (Slonimsky 1947, vi). These are a few of many options, but they serve to illustrate a fairly obvious, but nonetheless important, notion; as with all scales, a multi-octave scale may be harmonized by notes pertaining to that same pitch set—i.e., diatonically—or not. A perfect example of the latter option can be found in the opening passage of Elliot Carter's *Duo for Violin and Piano*.

In this work, Carter makes use of twelve-tone aggregates, "not as a source of rows to form melodic or harmonic shapes, but as tone structures that determine the precise octave placement of any given pitch" (Derby 1981, 153). The aggregate on which the violin part is based in the first 83 bars, spanning four octaves, is in effect a two-octave symmetrical scale based on the repetition of a four note subrow at intervals of a minor sixth, thus dividing the two-octave span into three equal parts. This two-octave structure is repeated, covering a total range of four octaves (see Figure 21). The scale corresponds to nº 698 in Slonimsky's *Thesaurus* (see Figure 22), though this is not to say that Carter necessarily obtained the scale from said source. In this particular passage of Carter's composition, the violin adheres to the notes contained within the two-octave scale, whereas the piano notes, when these lie in the same range as the violin, are taken from pitches not included in the violin's fixed octave scheme, acting as a contrast rather than a duplication of the violin's octave placements of pitches (Derby 1981, 153).

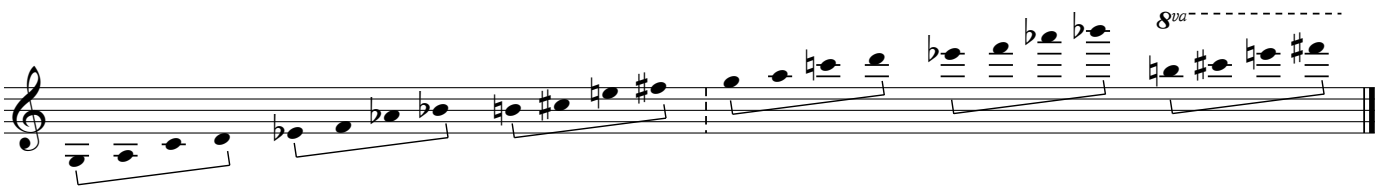

Figure 21 – Scale on which the violin part is based in bars 1—83 of Elliot Carter's Duo for Violin and Piano
Reference: Own elaboration after Derby (1981)

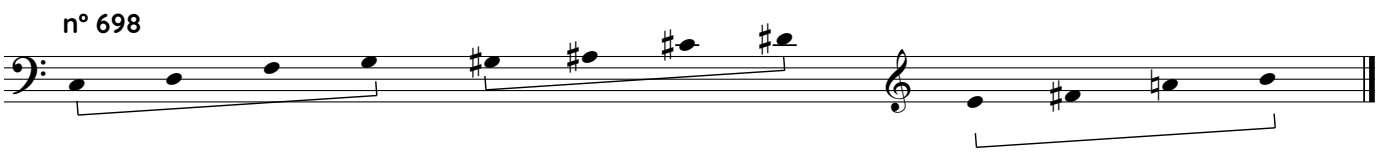

Figure 22 – Scale No 698
Reference: Own elaboration after Slonimsky (1947)

As illustrated by Derby's analysis, Carter uses other fixed octave schemes throughout the piece to organize its tonal content in different ways; whilst in the opening passage the two instruments do not draw on the same collection of pitches, in bars 150—63 both instrument parts consist of notes contained within the same fixed octave pitch set (Derby 1981, 153–155). Save for the two-octave scale in the opening passage, the other octave schemes used throughout the piece could be more adequately described as pitch fields, as opposed to scales, being restricted to a given intervallic range beyond which the structure is not repeated. Carter's Duo serves as a valid example of how music can be organized by pitch sets that fix notes in a given range, as opposed to being organized by pitch-class sets, as is the case with most music.

Regarding the use of these techniques, Derby argues that the repetition and reiteration of pitches within an atonal context in specific octave placements, brought about through the use of fixed octave schemes and characteristic intervals in place of tone rows, produces the kind of harmonic control which is often missing in atonal music (Derby 1981, 153–155). Carter is able to establish a distinct harmonic orientation within a given passage while allowing himself the use of all twelve tones by restricting recurring pitches to certain octave placements.

4.2. A single scale

While Carter uses different pitch sets to organize tonal content in his *Duo for Violin and Piano*, other pieces are completely or predominantly based on a single non-octave-repeating scale. This is the case with several compositions by Iannis Xenakis, who often employed large scalar structures that avoid periodicity at the octave from which to draw the tonal content in his work (Squibbs 1996, 60). *Komboï* (1981), written for harpsichord and percussion, is based entirely on a single non-octave-repeating scale, spanning six octaves, with few exceptions of non-diatonic pitches being employed throughout the piece (Pace 2001, 129). As noted by Pace (2001), "the use of the same scale, or filters thereof, provides a very marked harmonic consistency. The one exception to this rule (cr 313-319) is all the more sharply contrasted as a result" (131). The scale is shown in Figure 23.

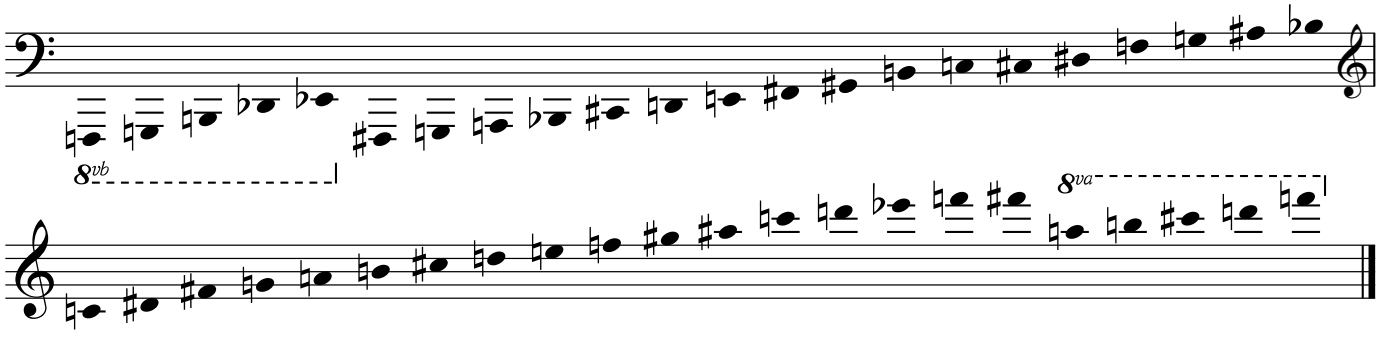

Figure 23. Scale used in *Komboï*
Reference: Own elaboration after Pace (2001)

Joel Hoffman also employs multi-octave scales in several compositions, often using a single formation to provide the pitch content for an entire piece, as in the case of works such as *6-8-2-4-5-8* (D. W. Brown 2019), *String Quartet* (Hoffman 2005) or *Piano Concerto* (Hoffman 2008). Hoffman's strategy for devising multi-octave scales, discussed in section 3.2.2, is very different from that of Xenakis. Hoffman's scales are also complex constructs spanning a large intervallic range, but, unlike Xenakis's formations derived from sieve theory, they are based on the repetition of a scale fragment throughout various octaves and

are thus symmetrical. Hoffman describes the effect of composing with such structures in the program notes to *Piano Concerto*:

> The effect of this harmonic structure is to give the piece a sort of musical "fingerprint" or overall color quality, a bit like the difference between the color of major or minor but on a much more subtle level. I'd like to think that all of my pieces composed with this method (each of which has a uniquely invented scale) have such a singular fingerprint. (Hoffman 2008)

4.3. Modulation and transposition

Whilst it is perfectly possible to base an entire composition around a single multi-octave scale, these constructs also lend themselves to transposition and some composers have exploited this possibility in order to structure and develop a piece of music. Weston considers the prospect of working in a chromatic context—many non-octave-repeating scales contain the full chromatic aggregate— whilst not forgoing the benefits concerning structure and modulation inherent in traditional scales, to be one of the principal assets of using non-octave-repeating scales (Weston 2020, 13). As with the scales contained in Slonimsky's *Thesaurus*, Weston suggests constructing non-octave-repeating scales using generative interval cycles that do not conclude within a single octave, thus resulting in symmetrical scales of a varying modular span, depending on their generative interval cycle (Weston 2020, 5). The author then elaborates on how modulation can be used to develop a piece of music when working with such scales, likening the modulation possibilities inherent in multi-octave scales to those found in the tonal system of traditional Western practice. A favorable property of the major scale in this sense is the great variety in the number of common tones under transposition the scale offers. This is due to the fact that each interval within the scale occurs a unique number of times, known as unique multiplicity property, meaning that transposition by a given interval will render a unique number of common tones with the original key with respect to transposition by any other interval (Carey 1998, 15). The one exception to this in the major scale is transposition by a tritone, which yields the same number of common tones as transposition by a semitone, namely two. Therefore, the variation in the number of common tones under transposition in the major scale ranges from two to six. In the words of Weston, the scale offers "five different levels of intersection in terms of pitch-class content, or put another way, five different degrees along the nearness/distantness continuum in modulation space" (Weston 2020, 8). In the case of non-octave-repeating scales, however, the author argues that when examining the nearness/distance relationship in so far as common pitch content under transposition is concerned, one must focus on common pitches, as opposed to common pitch classes. This is due to the fact that some of these scales may contain the full pitch-class aggregate, meaning all transpositions will have the same pitch-class content, but will differ in terms of the placement of these pitches in pitch space.

As with all symmetrical scales, Weston's non-octave-repeating scales transpose onto themselves after a limited number of transpositions. In order to analyze the "nearness/distance relationship" of a scale's possible transpositions, Weston places them in the same intervallic range by extending them at the bottom and truncating them at the top allowing for easy comparison in terms of pitch content, in the same way one could compare the pitch content of a C major scale and a Bb major scale by contrasting C major with C dorian. Figure 24 contrasts the eight possible transpositions of a three-octave scale with the scale in its original form. The open noteheads represent the pitches in common with the original scale and the notes with stems show the pitches belonging to the generative interval cycle, in this case, a major sixth. Although the scale contains

all twelve pitch classes, the difference in pitch content in each octave is notable from one transposition to another. Thus, for Weston, these scales allow one to compose in a highly chromatic context while still maintaining the advantages regarding structure and modulation found in composing with traditional scales:

> Unlike conventional octave-repeating scales, many non-octave-repeating scales include the total chromatic (or a large subset of it) without having to transpose the scale, or put another way, without having to modulate. But, the possibility of modulation systems still exists, as it does not, in any meaningful way, in music based on the chromatic scale. (Weston 2020, 12–13)

The author provides examples of his own work in which he believes that the effect of modulation in a multi-octave scale context can be clearly observed, namely the pieces *Glancing Spirals* and *Intensity 8.5* (see Musical References).

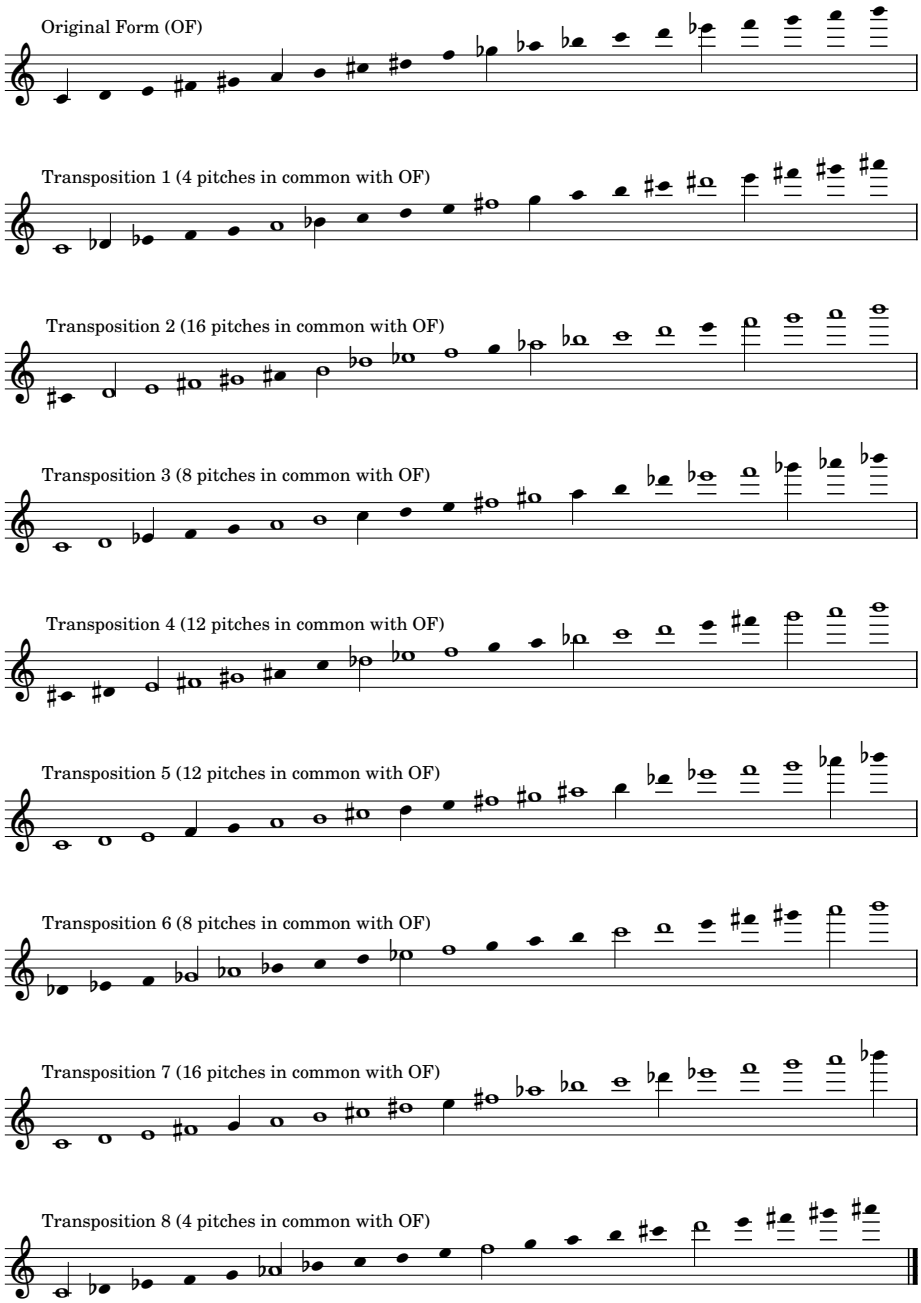

Figure 24. Common pitches under transposition. Reference: Own elaboration after Weston (2020)

In the work for solo piano *Mists* (1980), Iannis Xenakis also uses different transpositions of the scale on which the piece is based to organize pitch content. For this piece, the composer designed a 30-pitch sieve, spanning 90 semitones (Squibbs 2002, 92). The scale is shown in Figure 25. Unlike *Komboï*, where the pitch content is mostly based on the work's scale in its original form (Pace 2001, 129), Xenakis employs 11 of the 90 feasible transpositions of the scale in Figure 25 to organize the composition's tonal content (Squibbs 2002, 95). The composer applies procedures characteristic of his work developed from mathematical or graphic concepts to generate musical ideas and textures within the scale, such as *continuous* and *discontinuous walks*—referring to linear and non-linear movement—or *arborescenses*—quasi polyphonic configurations based on branching structures (Squibbs 2002, 116).

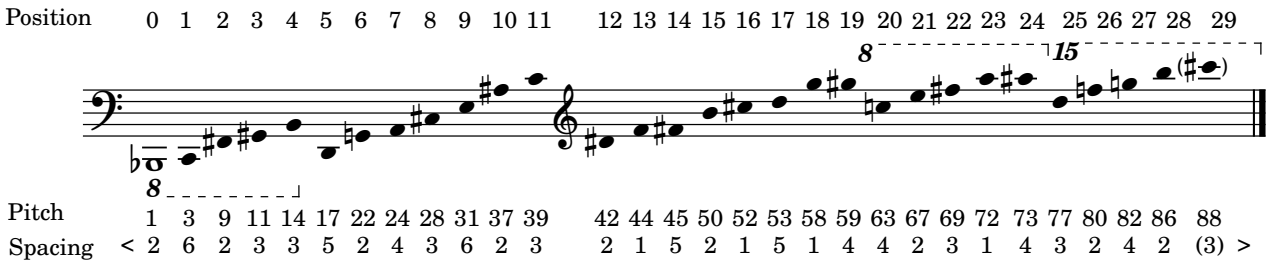

Figure 25 – Scale used created by Iannis Xenakis for the work Mists
Reference: Own elaboration after Squibbs (2002)

4.4. The Russian Dodecaphony

Elements of the Znamenny scale discussed in section 2.2 appear in the works of Russian composers such as Shostakovich, who often based his themes on elements of this mode after 1936 (Kirkman and Ivashkin 2016, 28), Mussorgsky, in the second theme of *The Great Gate of Kiev* (Nikolsky 2016) or Rimsky-Korsakov, for instance, in his *Russian Easter Overture* (Dubinets 2015, 322). An interesting development of the Znamenny scale in twentieth century music can be found in the work of the Russian composer Yuri Butsko, who developed it into a large multi-octave construct.

Butsko extended the Znamenny scale's range of a twelfth by stacking tetrachords below and above, thereby including the full pitch-class set, as can be seen in Figure 26. The open noteheads indicate the four original tetrachords of the Znamenny scale. Being based on an archetypal Russian concept and at the same time including all pitch classes, as with a serialist twelve-tone row, Butsko described this system as "Russian dodecaphony" and employed it for the first time in his *Polyphonic concerto* (1969) for piano, organ, harpsichord and celeste (Dubinets 2015, 323). Butsko's Russian dodecaphony influenced a generation of Russian composers who have made use of this concept, including Alexander Wustin, Mikhail Kollontay and Nikolai Korndorf (Dubinets 2015, 324).

The most famous work in which Butsko's expanded Znamenny scale is featured is in all likelihood Alfred Schnittke's *Symphony №4*, leading to the composer on occasions actually being credited for inventing the concept (Taruskin 2008, chap. 35). The piece draws its pitch content from three distinct non-octave-repeating scales, including Butsko's concoction, and provides an excellent example of their application in through-composed music.

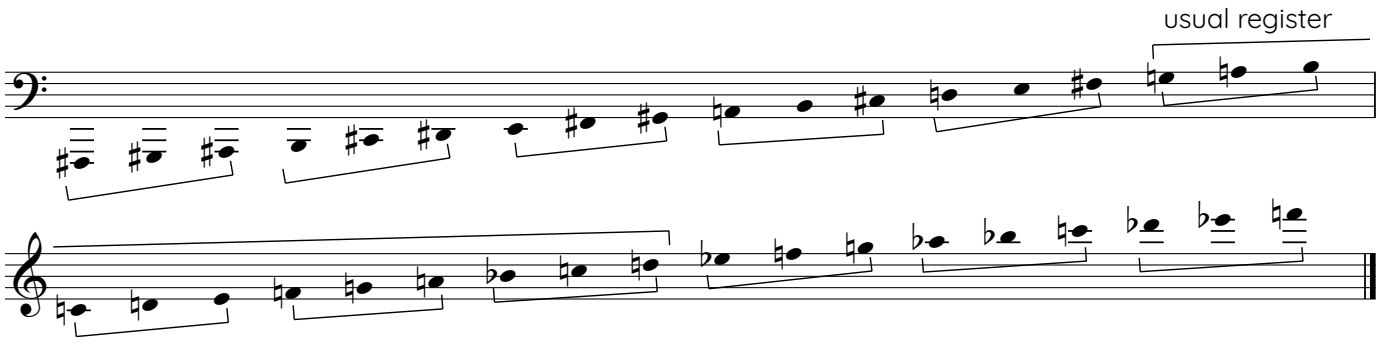

Figure 26 – Butsko’s extended Znamenny scale
Reference: Own elaboration after Lehmann (2018)

5. Compositional procedures employing non-octave-repeating scales in Schnittke’s Symphony №4

Drawing its pitch-content almost exclusively from non-octave-repeating scales, Schnittke’s Symphony №4 serves as a perfect example of how these structures may be employed in composition. The present section provides insight into some of the procedures employed throughout the piece concerning the application of the non-octave-repeating scales. For an in-depth analysis of the piece as a whole, the reader may refer to Gavin Dixon’s excellent work on the subject (2007).

5.1. Harmonic development

Since in scales of this nature pitch content varies in different octaves, augmented and diminished octaves occur when a given scale step differs from one octave to the next—e.g., B3 and Bb4 or F#3 and F4 in Butsko’s expanded Znamenny scale (Mode 1 in Figure 27). The presence of diminished and augmented octaves is the common feature amongst the harmonies employed in evoking the different religious cultures, creating what Schnittke refers to as a “distorted international space,” which is maintained throughout the work until the coda, where a diatonic scale is established through a combination of the different themes previously employed, using “corrected” pure octave (Schnittke 2002, 47).

As far as specific harmonic procedures that make use of the non-octave-repeating nature of the scales are concerned, the following are especially prominent throughout the work:

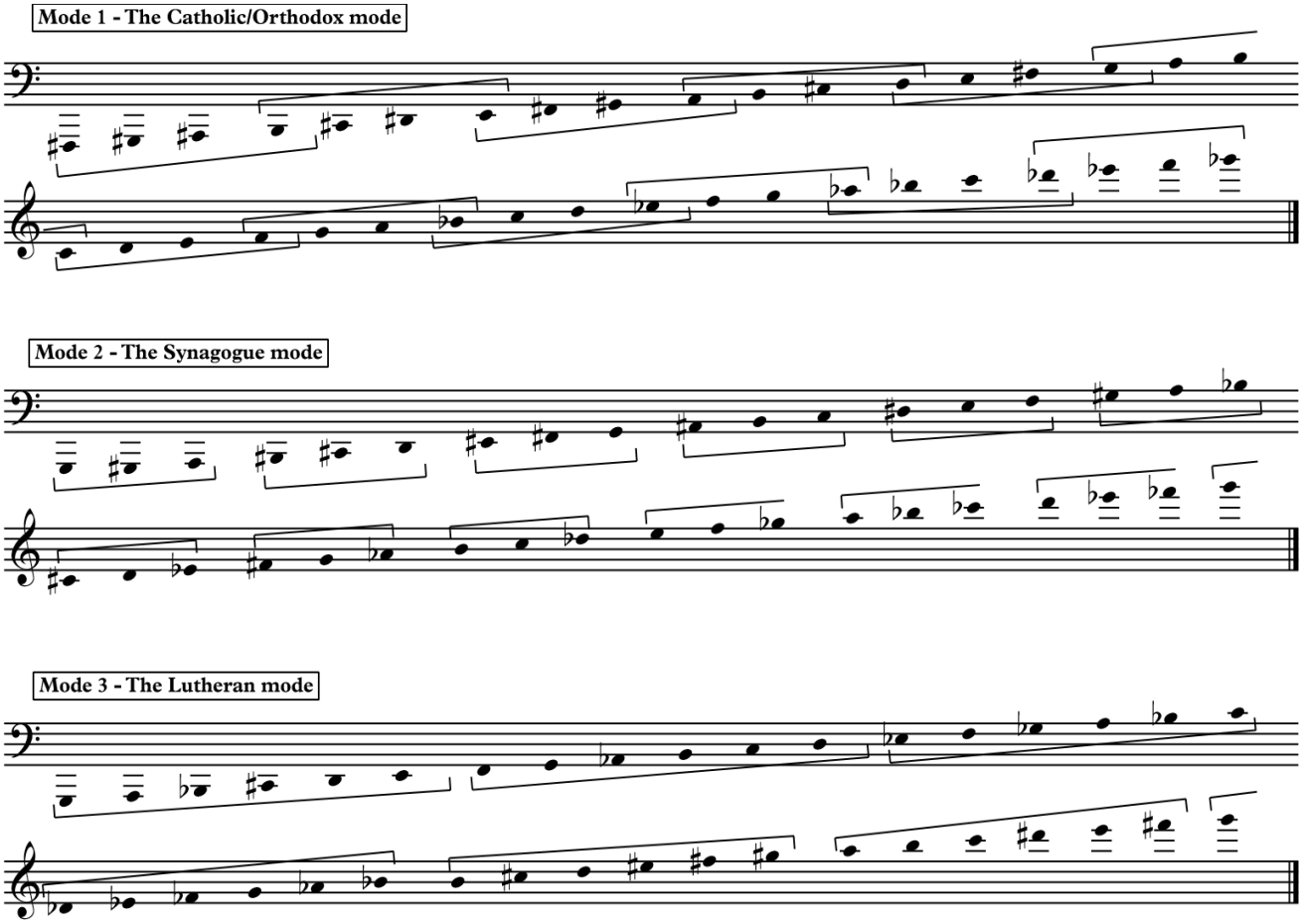

Figure 27 – Non-octave-repeating scales in Schnittke's *Symphony No. 4*

5.1.1. Chords based on diminished and augmented octaves

Chords resulting from octave doublings, which will be referred to as *octave-chords* from now on, are recurrent throughout the symphony. Due to the non-octave repeating nature of the scales, these chords consist of both perfect as well as diminished and augmented octaves—equivalent to the intervals of a major seventh and a minor ninth, respectively. Figure 28 shows the *octave-chords* used in the piano part in section B of the piece, based on Mode 2 in Figure 27, as well as a phrase from the same section demonstrating how they are applied. Frequently throughout the work, melodies are harmonized in this fashion (G. T. Dixon 2007, 175), usually moving through the respective scales in stepwise motion.

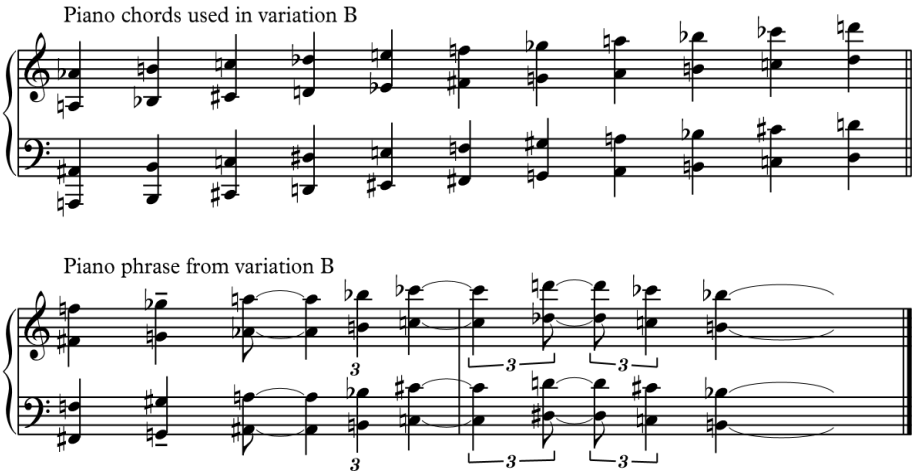

Figure 28 – The Synagogue mode harmonized with four-part *octave-chords*

For instance, in the fragment shown in Figure 29, taken from section U, *octave-chords* account for the pitch content in all instrument sections. The omnipresence of this sonority throughout most of the symphony makes the moment when perfect octaves occur in the coda even more effectively.

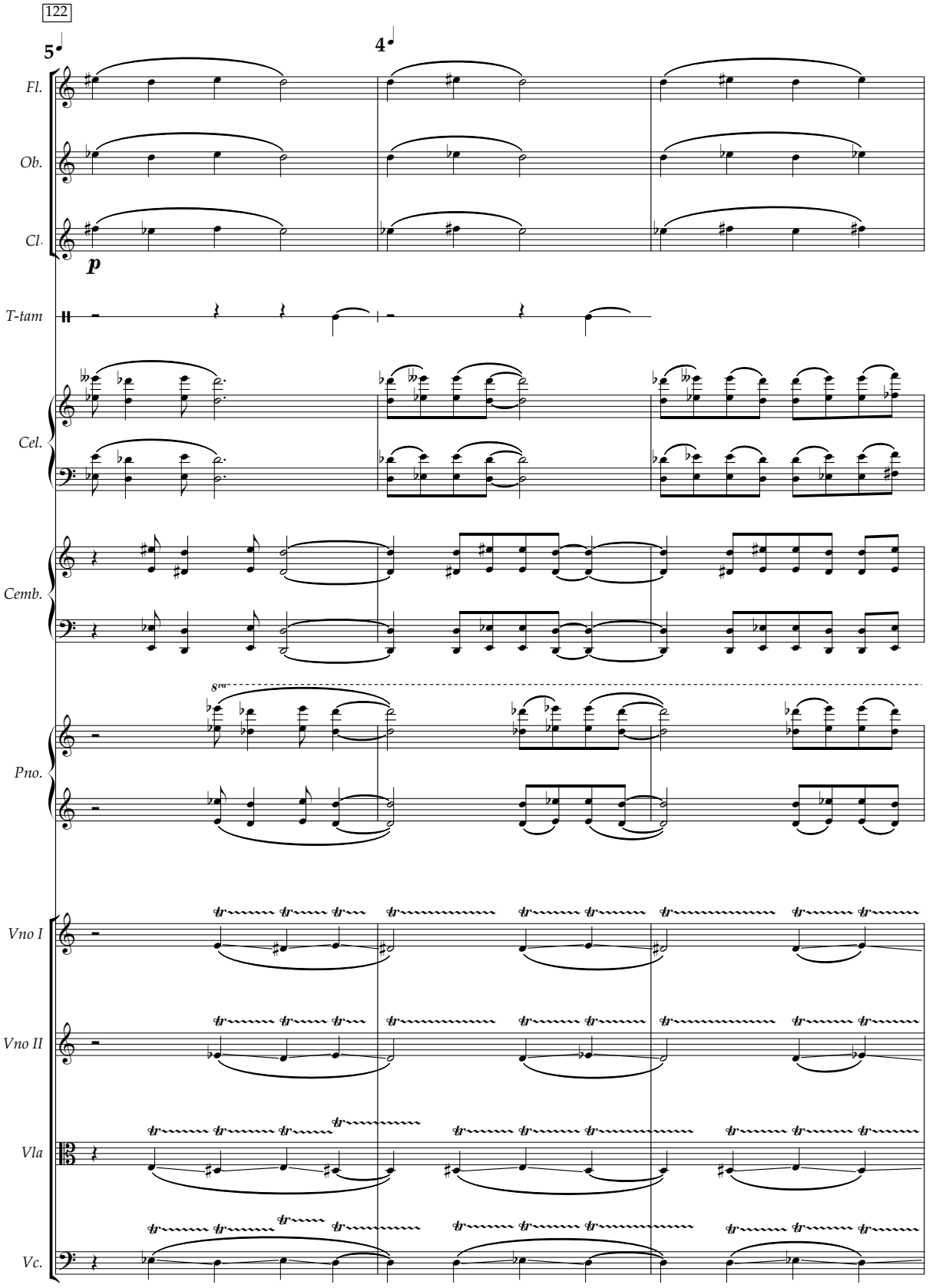

Figure 29 – Fragment based entirely on *octave chords*

5.1.2. Parallel movement

Schnittke makes use of parallel movement employing other chordal structures, as well as the *octave-chords* previously discussed, such as quartal or cluster cords. The phrases extracted from the piano solo in section G, shown in Figure 30, are a clear example of this. The differences in pitch content in neighboring octaves are sounded in close proximity, yet not simultaneously, meaning that the non-octave-repeating nature of the scale is expressed in the phrases, while not necessarily creating a high level of dissonance.

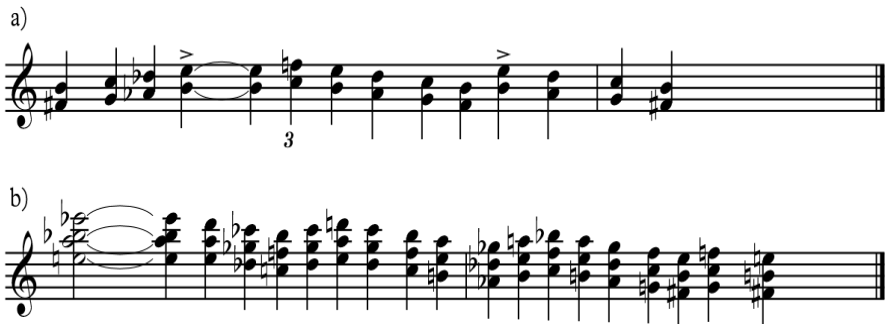

Figure 30 – Extracts from section G showing parallel movement of quartal chords within the Synagogue mode

5.1.3. Non-Octave-doublings of chordal structures

Chords that span a wide intervallic range are more likely to bring out the non-octave-repeating particularities of these scales. In several passages Schnittke constructs vertical structures with a multiple octave range by "doubling" chords in different sections of the scale at the diminished or augmented octave. Figure 31 shows two examples of this type of vertical structure taken from section G, based on the Synagogue mode, in which the composer makes extended use of such constructs. The cluster chord in Figure 31 *a)*, is the result of doubling the chord in the left hand, starting on G#3, two octaves higher in the right hand. Yet, due to the non-octave-repeating nature of the scale, the chord in the upper register starts not on G#5, but a semitone higher on A5, at the augmented octave. The same occurs with the three-part quartal chords in Figure 31 *b)*. This procedure exploits the non-octave-repeating nature of the scale by simultaneously sounding the varying pitch content in different ranges of the scale in order to construct decidedly dissonant structures which are highly effective, given the intensity of the passage in question.

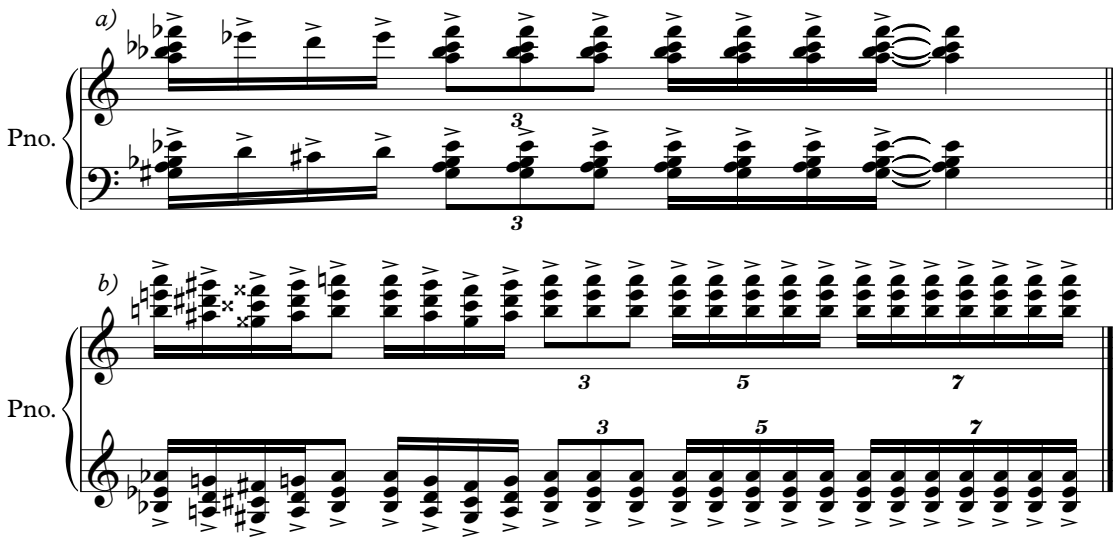

Figure 31 – Excerpts of the piano solo in Section G: chord doublings at the augmented double octave

5.2. Thematic development

The thematic development in Schnittke's *Symphony №4* is based largely on repetition and variation of melodic cells and rhythmic motifs (Schnittke 2002, 47) and exploits the diminished and augmented octaves inherent to the multi-octave scales from which the work's pitch content is derived in several ways. In many passages, a rich texture is achieved by displacing motifs at regular rhythmic intervals, causing them to overlap in what Emilia Ismael-Simental describes as a perpetual canonical structure (Ismael-Simental 2016, 34). There are three recurring ways in which the composer develops these motifs within the different multi-octave scales:

5.2.1. Intervallic transposition

Motifs are transposed several times at the distance of an interval at which the scale repeats, meaning that the motif maintains its exact intervallic structure, as would be the case with octave-transposition in a standard octave-repeating scale. Yet, since the scales used in the work do not repeat at the octave, each time the motif is transposed its pitch content differs. The development of motifs within the Synagogue mode in the woodwinds and strings in section K is a perfect example of this procedure. The motifs are repeated at the interval of a minor seventh, which is the closest possible to octave transposition given the scale's construction.[9] Figure 32 shows how this procedure is applied to the first two motifs in section K in the strings.

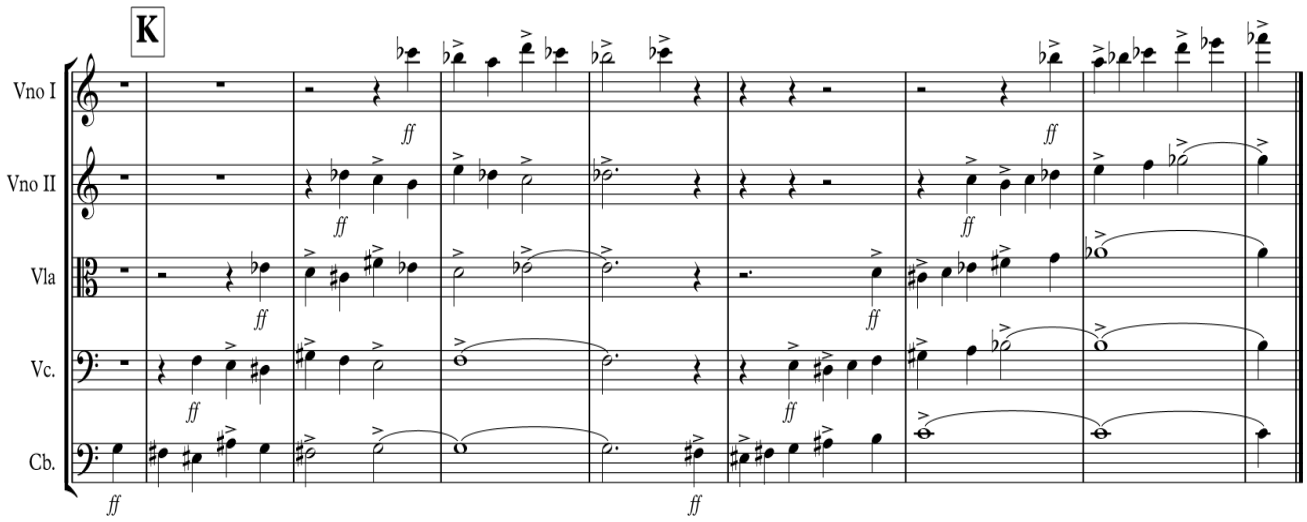

Figure 32 – Excerpt of section K of Schnittke's *Symphony №4*

5.2.2. Transposition at the octave

Motifs are transposed within the scale at the interval of an octave. Although the melodic contour of the transposed motifs is maintained, their actual pitch content varies in successive octaves, due to the non-octave-repeating nature of the scales; at a certain point this will naturally also affect the motif's starting note. Such is the case with the semi-quaver motif in the keyboard instruments and strings which provides the background throughout section F (see Figure 33), based on the Catholic/Orthodox mode in Figure 27. The three-note phrase is repeated in different octaves, commencing on B or Bb, depending on the pitch

[9] Since the scale is comprised of similar tetrachords at the distance of a perfect fourth (see Figure 27), a motif can be transposed within the scale maintaining the exact intervallic contour not at the octave, as would be the case in a standard octave-repeating scale, but at the perfect fourth and therefore at the minor seventh, being the sum of two perfect fourths.

content in each given octave. Since the scale is non-octave-repeating, each phrase, while sharing the same melodic contour, is intervallically different. Figure 34 shows the sections of the scale on which the different motifs occur.

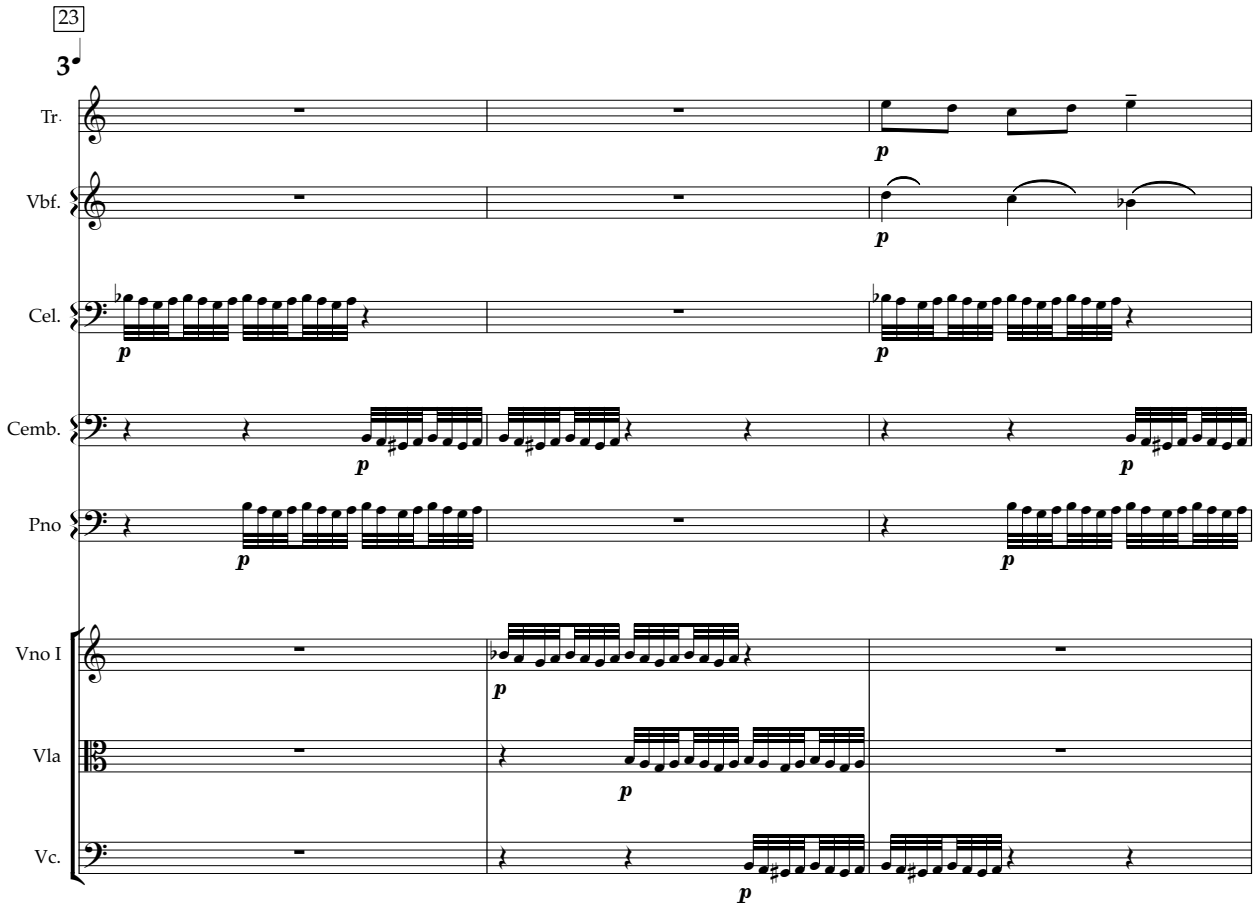

Figure 33 – Transposition of a motif at the octave

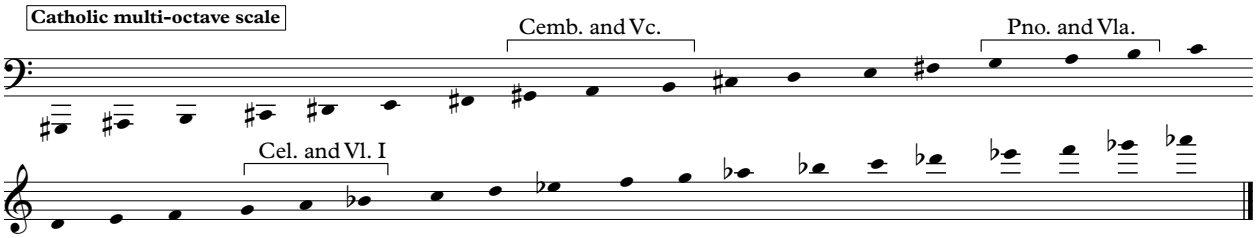

Figure 34 –Sections of Mode 1 used in the motifs in Figure 33

5.2.3. Rhythmic displacement within the same register

Motifs are repeated in a canonical fashion by different instruments in the same octave. In section E, the motifs in the oboe are echoed in the same octave two beats later, first by the flute, then by the clarinet. Likewise, the motifs in the first violins are repeated by the second violins, then by the violas (see Figure 35). In this case, the non-octave-repeating nature of the scale is less apparent in the individual instrument sections, given that the motifs are repeated within the same octaves, but is very noticeable in the overall texture, due to the difference in register between the woodwinds and strings, meaning that the varying pitch content in the scale's different octaves sounds continuously throughout the passage.

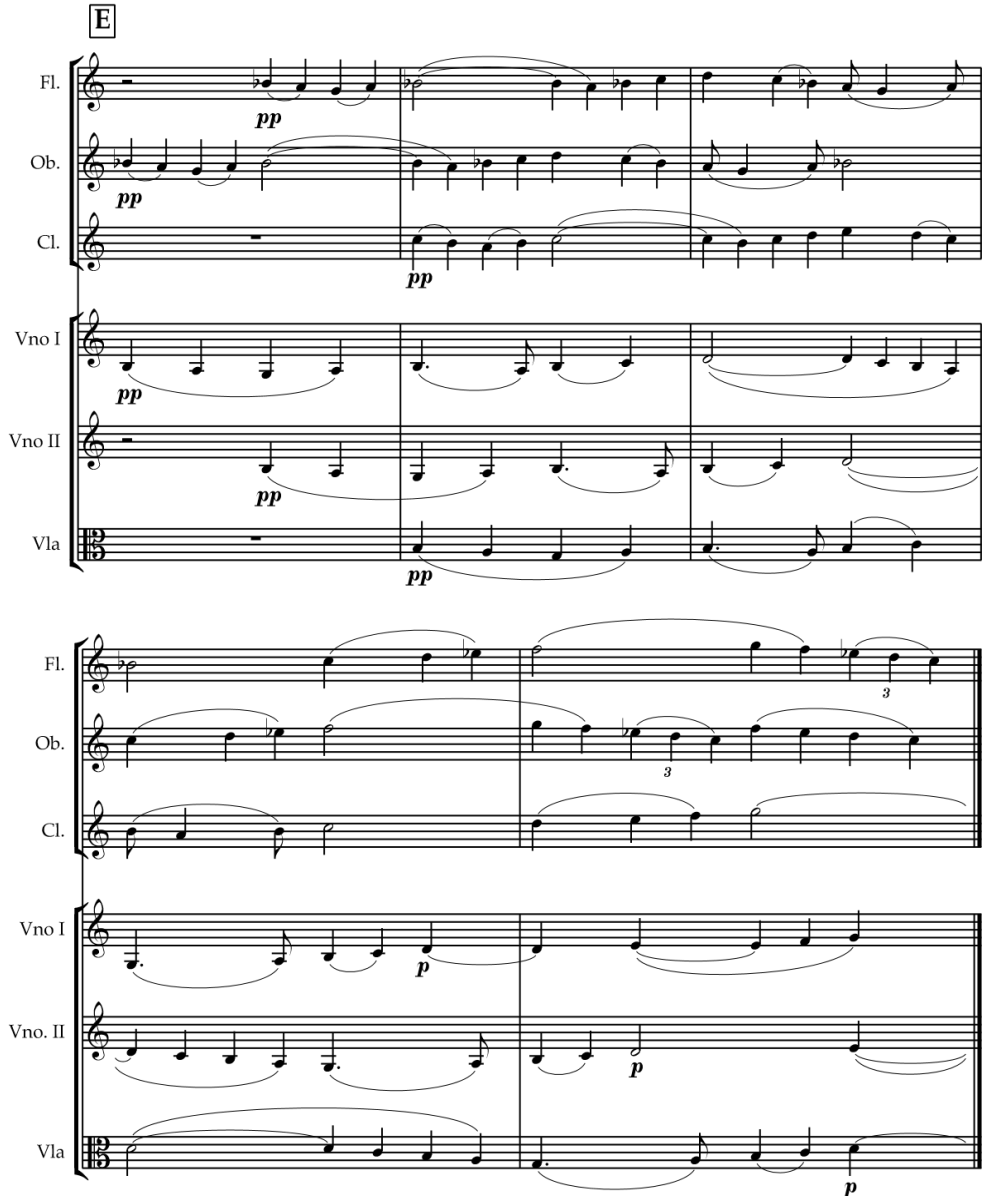

Figure 35 – Section E of Schnittke's *Symphony №4*

5.3. Simultaneous use of different multi-octave scales: Multioctave scale polymodality

While some sections of Schnittke's *Symphony №4* are diatonic to a single multi-octave scale, in others the composer employs a polymodal approach, using different multi-octave scales simultaneously by assigning each to a specific instrument or instrumental section. An overview of the distribution of the three scales in the different instrumental sections throughout the work can be found in Dixon's analysis (2007, 176).

Section I provides an especially interesting way of combining the three scales which, to our knowledge, has not been discussed in depth in the previous work dealing with this piece. As in many sections of the work, motifs occur in a consecutive and overlapping manner, on this occasion in the three keyboard instruments. Each instrument draws its tonal content from a different multi-octave scale. At first glance, there does not seem to be a definite and clear unifying concept common to the chords used throughout the passage, which are of varying intervallic construction. However, upon closer examination, one finds that the chords in the three instrument parts, though structurally very different, share the same pitch content. This can be observed by comparing the seven chords which constitute the first phrase of section I as they appear in each

keyboard instrument (see Figure 36). Since each part is based on a different multi-octave scale, the pitches that constitute a given chord are placed in the octave in which they are available in each scale. Take the G# in the first chord of the opening phrase in Figure 36 which appears as G#3 in the piano part—based on the Synagogue mode—, G#2 and G#4 in the cembalo part—based on the Lutheran mode—and *G#*5 in the celeste part—based on the Catholic/Orthodox mode. By comparing the chords in Figure 36 with the different multi-octave scales in Figure 27, one can see how the rest of the pitches in the different chords follow the same logic.

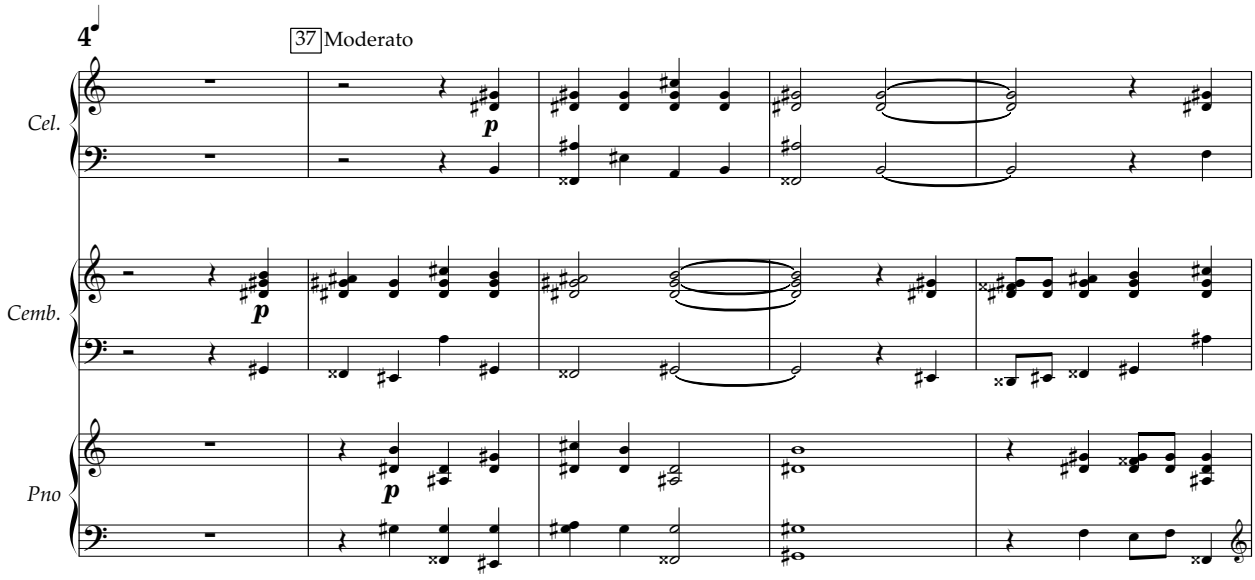

Figure 36 – Opening phrase in keyboard instruments in section I

Placing a reduced pitch-class set—that which constitutes each chord—in a different register depending on which multi-octave scale the part is based on, accounts for the intervallic variety of the chords in this section. In order to understand the construction and the logic behind the individual chords, it is necessary to place the notes within the different phrases in a similar register to analyze the voice leading. Remarkably, the result is a melody moving predominantly in stepwise motion, harmonized by parallel major and minor thirds, against a double pedal-point, most likely reminiscent of medieval liturgical music. Figure 37 shows the first two phrases of section I as they appear in the different keyboard instruments and a reduction placing the individual voices in a similar register with close voice leading.[10] For the sake of comparison, the phrases are displayed without the rhythmic displacement that occurs in the actual piece. Note that the top voice in the reduction is diatonic to the Lutheran mode and the melodic theme is the one associated with the Protestant faith throughout the symphony (G. T. Dixon 2007, 169). The pitches that constitute the theme as it is displayed in the reduction are sounded in the cembalo part, which is based on the Lutheran mode, yet not always in the top voice. This theme can be clearly heard in section P, first in the piano part in the same register as in section I, and then in the alto part in a different register.

[10] The enharmonic spelling of pitches in the original parts is respected.

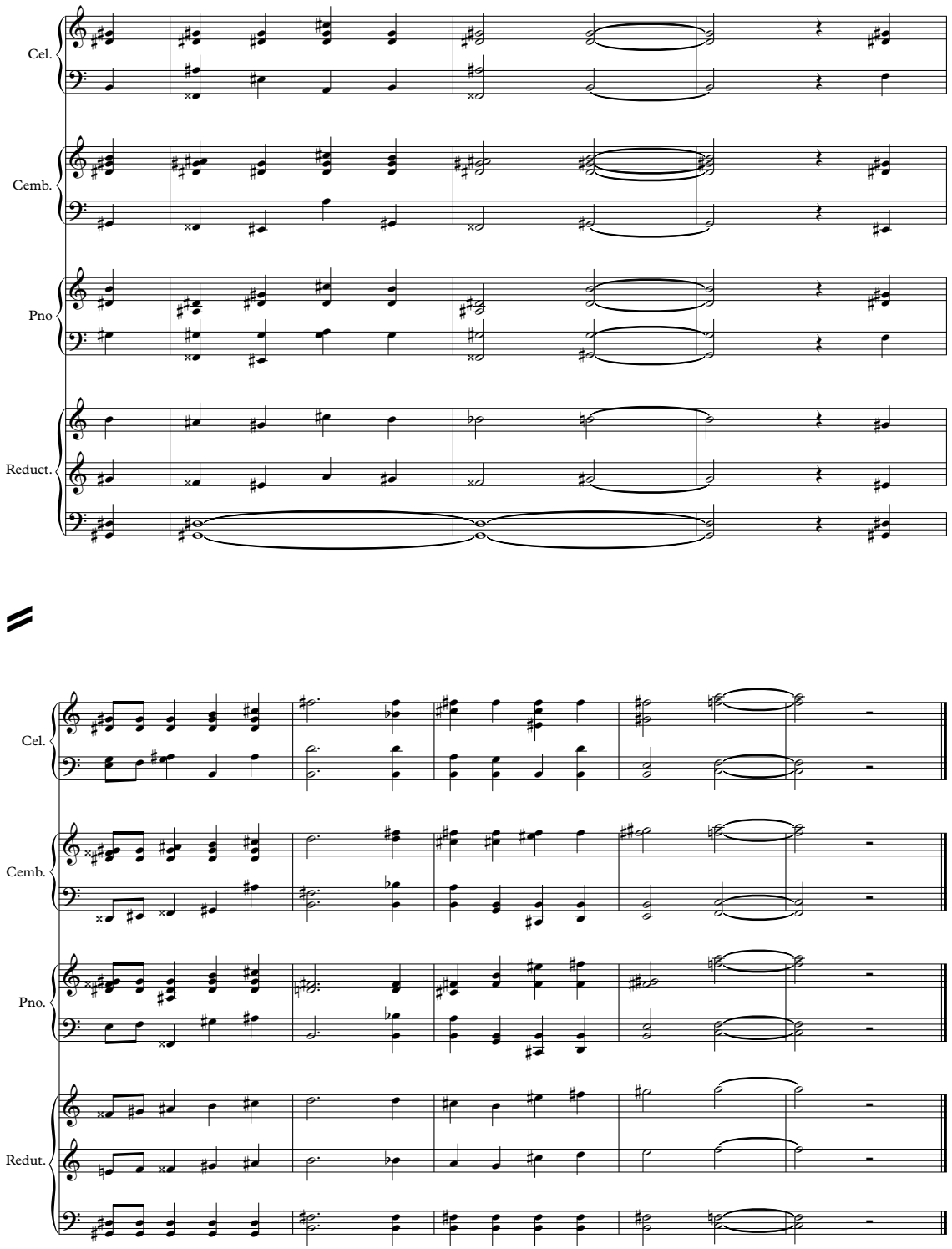

Figure 37 – First two phrases in section I as they appear in the different keyboard instruments and the reduction in a similar range

6. Application in jazz or improvised music

Although the use of non-octave-repeating scales within the practice of jazz or jazz-related music is far from common, there have been musicians who have applied such structures to expand their musical palette. When doing so, the usage of non-octave-repeating scales tends to serve a specific purpose, like playing over

polychords or playing *outside*.[11] Pieces totally or even partially governed by specific multi-octave scales in this context are extremely rare. This section presents an overview of how certain jazz musicians have employed non-octave-repeating scales in their work.

6.1. Multi-octave scales for playing over polytonal chords

Jazz educators such as Dennis Sandole, David Baker or David Liebman suggest assembling non-octave-repeating scales through the combination of strings of notes pertaining to different key centers in order to improvise over polytonal chords or for playing *outside* in a process analogous to Persichetti's multi-octave scales constructed by a succession of similar and dissimilar tetrachords, in the sense that a large scalar structure is assembled by connecting smaller units; see McGill (2013), Baker (1990), and Liebman (1991). These scales vary in intervallic range and, whilst they generally span over an octave, do not always conclude at a given octave. For instance, the C/Gb scale in Figure 38 spans an augmented 11th, falling thereby a tritone short of octave-conclusion, suggesting that its modular interval span is not necessarily intended to repeat throughout the entire feasible intervallic range, as is generally the case with scales.

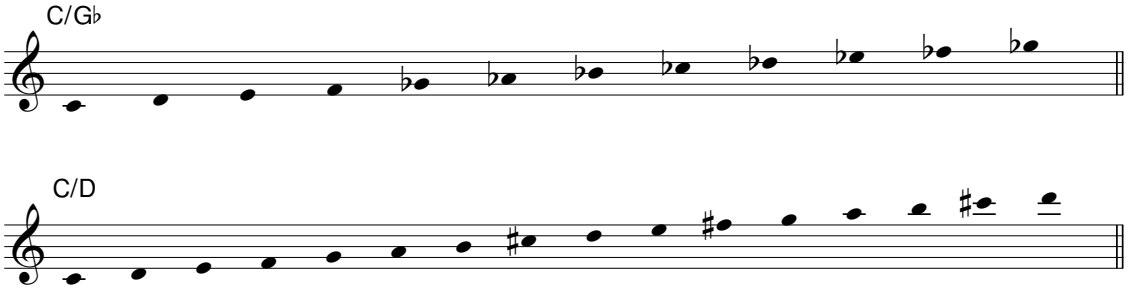

Figure 38 – Examples of bitonal scales
Reference: Own elaboration after Baker (1990)

An example of the usage of such structures can be observed in the analysis of Liebman's composition *Carissima*, based on polytonal harmonies (Liebman 1991, 131). Various possible scale options are suggested for each of the five polychords constituting the piece's chord progression, most of which are not octave-repeating, the largest comprising a major 17th in range (see Figure 39).

[11] A term used to refer to when improvisors play material unrelated to the underlying harmony, generally as a way of increasing harmonic tension.

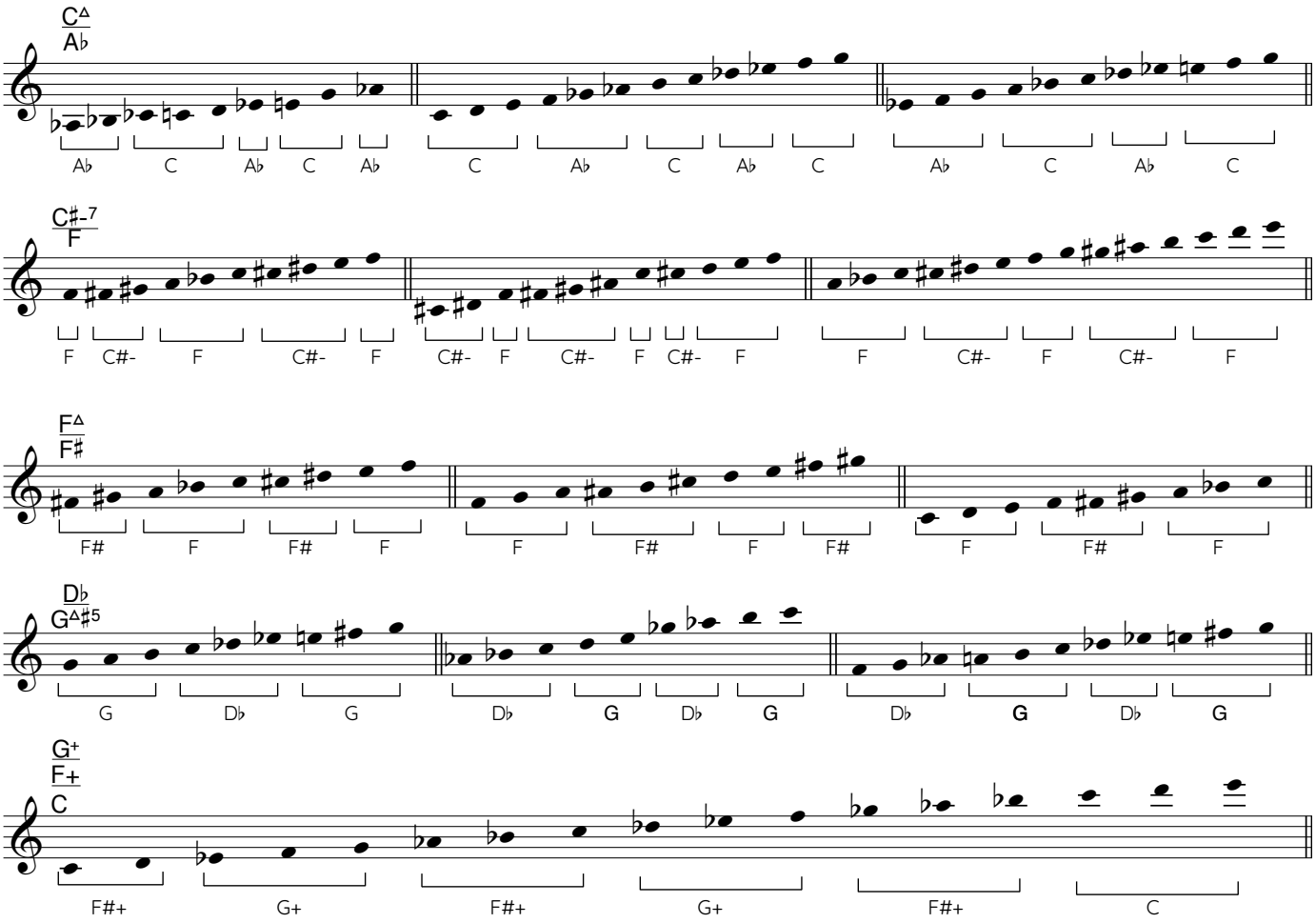

Figure 39 – Synthetic scales suggested for improvising over the chord progression in *Carissima*
Reference: Own elaboration after Liebman (1991)

6.2. Anders Lønne Grønseth: The Bitonal Scale System

Whilst in the previous examples of the application of multi-octave scales in a jazz context the scales are conceived as a possible approach for a specific purpose, such as improvising over polychords or for playing outside, the Norwegian saxophonist Anders Lønne Grønseth has explored the possibility of using multi-octave scales more extensively as a form of tonal organization and details his approach in The Bitonal Scale System—a draft for a modal-harmonic system, available on his webpage (Grønseth 2015).

6.2.1 Construction

Grønseth constructs multi-octave scales by combining two or more standard one-octave scales, but, as opposed to linking successive scale fragments in the fashion of Baker or Sandole, has developed a personal manner of achieving this, where the original scales on which the resulting multi-octave scale is based remain less immediately apparent. The author arranges the scales which are to be combined in thirds, rather than in seconds, creating two-octave arpeggio-like structures, which, when combined, result in a two-octave scale. Figure 40 shows the two-octave scale resulting from the combination of the scales B and C major in this manner.

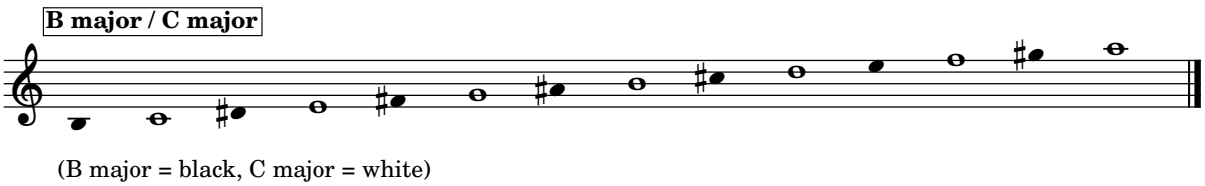

Figure 40 – C major and B major arranged in thirds in order to create a two-octave bitonal scale
Reference: Own elaboration after Grønseth (2015)

Grønseth labels the combination of two identical scales of different transposition a minor second apart as *bitonal leading note scales*, making the scale shown in Figure 40 the *B / C major bitonal leading note scale*. Although Grønseth's subsequent analysis and observations concerning the nature and application of bitonal scales all relate to this particular scale, he also suggests combining three or more different scales to achieve a two-octave scale and provides various examples.

6.2.2. Key centres

After dealing with the construction of bitonal scales, Grønseth provides insight into how these may be analyzed and understood, as well as offering ideas on how they may be developed. An interesting notion is introduced concerning a bitonal scale's tonal behavior, namely that, since the pitch-class content differs in subsequent octaves, tonal centers will be formed where a pitch is present in both octaves, thus forming a perfect octave. In the case of the *B / C major bitonal leading note scale* used as an example throughout the text (see Figure 40), there are two such pitches—B and E—, which the author suggests can be considered as the *root (primary key center)* and *dominant (secondary key center)* of the scale (Grønseth 2015, 5). Once the key centers have been defined, the author divides the scale into a succession of tetrachords.

> In a bitonal scale, tetrachords are defined by the key centres of the scale. When tetrachords on the key centres are established, one can locate the other tetrachords involved in the scale. In this way we get a row of tetrachords which together constitute the leading note scale. On each starting note of a tetrachord, a *tonal centre* is formed. (...) The *skeleton* of the bitonal scale is defined by the tonal centres of the tetrachords which it is constructed from. (Grønseth 2015, 8)

Figure 41 shows this process using the *B / C major bitonal leading note scale* from Figure 40 as an example, now labeled *E lydian / F lydian* since the note *E* has been established as the tonic.

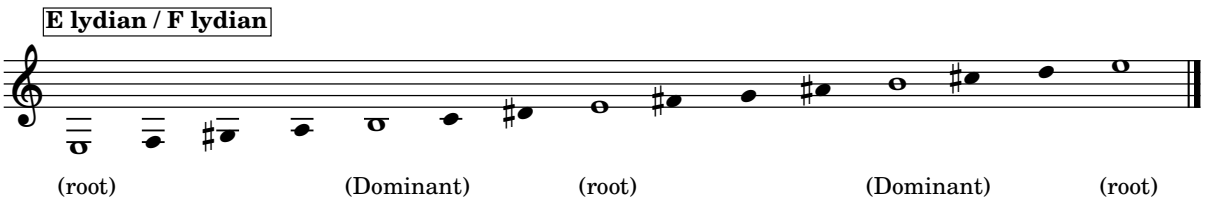

Figure 41 – Tetrachords and key centers in a Bitonal scale
Reference: Own elaboration after Grønseth (2015)

The notion that a non-octave-repeating scale will naturally gravitate towards the pitches that repeat at the octave is perfectly logical, considering the stability of the interval due to the effect of octave-equivalence. The concept of establishing primary and secondary key centers on these pitches and subsequently dividing the scale into different tetrachords works perfectly in the case of the *B / C major bitonal leading note scale*

used in the example, in which only two pitches are repeated in both octaves, yet how it would function in a scale with a higher level of octave repetition—such as the *C major / C harmonic minor* (see Figure 42), in which five different pitches repeat at the octave—is not discussed.

Grønseth points to ways in which this material could be developed—such as through the permutation of the different tetrachords that form a given bitonal scale—and offers examples of vertical structures derived from bitonal scales, but has as of yet, to our knowledge, not written the second part of his text, which is to deal with the practical application of what the author has labelled the Bitonal Scale System.

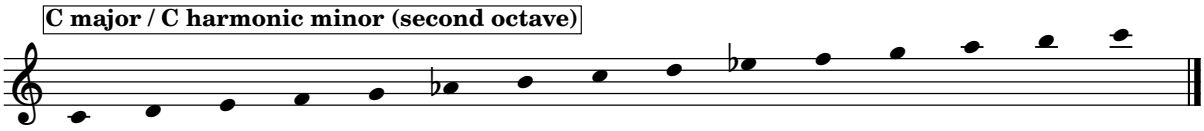

Figure 42. C major / C harmonic minor scale
Reference: Own elaboration after Grønseth (2015)

6.3. The improvised cadenza in Hoffman's *Piano Concerto*

Though not related to jazz music, a clear example of an improvisation being structured by a specific multi-octave scale can be found in the improvised cadenza in Hoffman's *Piano Concerto*, previously discussed in section 4.2, where the performer is to extemporize within the multi-octave scale on which the piece is based. The composer comments on this fact in the program notes:

> The instructions in the score state that the pianist is free to invent whatever he/she wishes, on condition that only the 68 notes of the piece are to be used. Presumably, preparing for this will produce a brand-new take on practicing scales and arpeggios! (Hoffman 2008)

Comparing the two versions of the improvised cadenza by pianist Edward Neeman—see Neeman-1 and Neeman-2 in the Musical References section—, one can clearly observe the range of textural possibilities improvising in such a context can yield. Hoffman's work is in effect a piece of music based on a multi-octave scale, in which composed and improvised elements coexist.

7. Discussion

Musicians have devised non-octave-repeating scales in different ways for specific musical purposes: Schnittke constructs the scales in *Symphony №4* from melodic motifs in order to further develop them; Xenakis obtains his scales from pitch sieves; Liebman, Baker or Sandole assemble their scales by linking tetrachords pertaining to different key centers to improvise over polychords. The manner and extent to which these structures have been used to organize pitch content varies from one composer to another and from one piece to another, as does their construction in terms of intervallic span, pitch content and complexity, depending on the composer's intentions regarding their application. A multi-octave scale may be employed in a particular passage, as in Carter's *Duo for Violin and Piano* or throughout an entire piece, as in Xenakis's *Komboï*. Different transpositions of a given scale may be used to provide harmonic structure, as in Weston's compositions, or only the scale in prime form, as in Hoffman's *Piano Concerto*. This last piece provides an example of a composition in which pitch content is drawn exclusively from a single non-octave-repeating scale, though other pieces based on similar scales do allow for extra scalar pitches, such as *6-8-2-4-5-8*, by the same composer. A composition may be governed by a single non-octave-repeating scale or by

several, as in Schnittke's *Symphony №4*. As far as the application of such scales in improvisation is concerned, a scale may provide the tonal content for an entire improvisation, as in Hoffman's *Piano Concerto*, be used as a tool for improvising over composite chord structures, as in Liebman's *Carissima*, or as a strategy for playing *outside* (McGill 2013).

This paper also illustrates that the extent to which non-octave-repeating scales have been exploited in through-composed music within Western Compositional Practice differs greatly from the extent to which they have been employed in jazz and jazz related music. In the case of the former, these scale constructs have been employed as essential structures with which to organize pitch content and composers have developed a variety of strategies in the process of their application. In jazz improvisation they have been given far less importance, being used for coloristic effects or to structure certain phrases, but not as a vital element of structural importance; a tool for playing *outside*, but not for defining what is *inside*, so as to speak. Or, as in the case of Liebman, one of a series of scalar options to accommodate horizontal structures for which standard octave-repeating-scales are unsuitable for, such as *add-on voicings* or polytonal chords. That being said, Grønseth's music based on his Bitonal Scale System points to a growing interest in such scales by jazz musicians and serves as an example of music in which non-octave-repeating scales play a pivotal role in organizing pitch content in both composition and improvisation in the context of jazz music. A further example of an improvisation within a specific multi-octave scale in Western Musical Practice is the improvised cadenza in Hoffman's *Piano Concerto*. As well as being a case in which a non-octave-repeating scale functions as a determining factor in organizing pitch content in the context of improvisation, the work also serves as a precedent of a piece based on such a scale in which composed and improvised elements coexist.

8. Conclusion

The overview provided in the present paper shows that non-octave-repeating scales can and have been effectively employed both in composition and improvisation, thus confirming that these structures are indeed a viable and time-tested form of organization. The account of their application in different cultures over several centuries, assembled by weaving together sources pertaining to different areas of musical knowledge, provides a narrative that brings forth the extent to which non-octave-repeating scales have been present all along. In doing so, we hope to bring the subject to the attention to fellow researchers and musicians, who may choose to employ these structures in their work or to undertake further research into the application of non-octave-repeating scales in different musical cultures.

Given the scarcity of literature dedicated to the subject, the possibilities for future research are abundant. For instance, with the exception of the analysis of Hoffman's *6-8-2-45-8* provided in D. W. Brown (2019), a thorough analysis of the works of musicians such as Gao Weijie, Joel Hoffman or Anders Lønne Grønseth, which rely heavily on non-octave-repeating scales as a form of tonal organization, is currently missing. Much could be learned from studying the techniques developed by composers such as these who have worked extensively with such scales. As pointed to in the discussion, with the exception of Grønseth, the use of non-octave-repeating scales as an organizational principle in jazz and jazz related music is extremely rare. This also opens the door to practice-led research that seeks to develop strategies for applying these structures in compositional and improvisational practice within the context of jazz music.

Another line of research is related to the perception and cognition of non-octave-repeating scales. Whilst, as previously discussed, certain composers have found the possibility of modulation within such scales to be advantageous, to which degree listeners are actually able to discern between diatonic and non-diatonic pitches—a necessary condition if modulation is to be meaningful—within a scale that contains the complete chromatic aggregate is yet to be proven empirically.

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