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The Roles of Mathematics in the History of Science: The Mathematization Thesis

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Abstract:

In this paper, we present an analysis of the evolution of the history of science as a discipline focusing on the role of the mathematization of nature as a historiographical perspective. Our study is centered in the mathematization thesis, which considers the rise of a mathematical approach of nature in the 17th century as being the most relevant event for scientific development. We begin discussing Edmund Husserl whose work, despite being mainly philosophical, is relevant for having affected the emergence of the narrative of the mathematization of nature and due to its influence on Alexandre Koyré. Next, we explore Koyré, Dijksterhuis, and Burtt's works, the historians from the 20th century responsible for the elaboration of the main narratives about the Scientific Revolution that put the mathematization of science as the protagonist of the new science. Then, we examine the reframing of the mathematization thesis with the narrative of two traditions developed by Thomas S. Kuhn and Richard Westfall, in which the mathematization of nature shares space with other developments taken as equally relevant. We conclude presenting contemporary critical perspectives on the mathematization thesis and its capacity for synthesizing scientific development.

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Introduction

One of the most well-known episodes of the history of science is the so-called Scientific Revolution. Different historians and philosophers have distinct views on the subject. Moreover, as a historiographical category, it has a history of its own. In the 18th century,

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scientists as Fontenelle, Lavoisier, Clairaut, D'Alembert, and Diderot had already used the term "revolutionary" concerning scientific works, even if sporadically (Nickles 2017; Cohen, I. 1985, 4). However, it is in the 20th century that the idea of the scientific revolution was integrated into historical narratives. The works of Eduard Jan Dijksterhuis and Edwin Arthur Burtt, both published in 1924, incorporated the idea of a discontinuous transition in scientific development. However, the term scientific revolution was coined only in 1935 by Alexandre Koyré (1892–1964) in three essays, lately gathered in his book *Études Galiléennes.* Historians of science diverge on their understanding of the duration, definition, and even on the revolutionary character of the so-called Scientific Revolution.[3] For instance, Butterfield (1965, 7-8) considers it as a foundational event of science, and a breaking point in human history. While Kuhn (2012, 4, 156) and Feyerabend (1993, 165-166) add the notion of incommensurability to the narrative.

The idea of rupture from an Aristotelian natural philosophy towards modern science is a stable factor in the majority of narratives of the Scientific Revolution. One of the main ingredients of such breakdown is the "mathematization thesis," also called "mathematization of nature," a term attributed to Koyré (Cohen, 2016, 143-148). The thesis claims that

> No other episode in the history of Western science has been as consequential as the rise of the mathematical approach to the natural world, both in terms of its impact on the development of science during the scientific revolution but also in regard to the debates that it has generated among scholars who have striven to understand the history and nature of science. (Gorham and Waters 2016, 1)

Galileo Galilei (1564–1642) is often portrayed as a precursor of the use of mathematics in the explanation of natural phenomena, breaking with a long Aristotelian tradition, which supposedly had rejected it. However, some contemporary authors problematize such a disruptive view on epistemological and sociological grounds. For instance, they consider that the mathematization impulse came more from the expansion of Aristotelian mixed mathematics than from the works of Galileo and his contemporaries (Schuster 2017, 48-65; Gingras 2001, 383). Besides, Galileo's work cannot be seen as definitive in the establishment of a consensus about the mathematical approach to natural phenomena, because there was significant resistance to this approach at least until the reception of Newton's *Principia* (Gingras 2001).

Although with different meanings, roles and emphasis, the mathematization thesis appears in the works of some important historians as Koyré, Dijksterhuis, and Burtt. In the present paper, we focus on their work due to their status as precursors of a contextual history of science (Cohen 1994, 88; 2016, 148), and for their consideration of the use of mathematics as a criterion of differentiation between the new science and the old scholastic philosophy.[4] We discuss the mathematization thesis as a historiographical category to analyze the studies of nature in the period of the Scientific Revolution. It is important to emphasize that the issues discussed here are contingent to the studies of mechanics in the 17th century, and would be different if our object of analysis were, for example, the history of the mathematization of electrical phenomena in the 18th and 19th centuries.

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[3] Some continuist historians, as Duhem, Randall, Crombie and Peter Dear, point to the dependence of the "revolutionary thinkers" on older works and traditions (Nickles 2017).
[4] It does not mean that they were the first to recognize the mathematization of nature as an important feature of the new science. During the 18th and the 19th centuries, philosophers and historians were already discussing the relations between mathematics and science, such as Mach (1903) and the thinkers from the Neo-Kantian school of Marburg (Heis 2018) that influenced both Koyré and Husserl.

We begin analyzing Husserl's thought because of his importance to the mathematization thesis, his influence over Koyré and his centrality to the emergence of the narrative of the mathematization of nature (Roux 2010, 319). Next, we explore the works by Koyré, Dijksterhuis, and Burtt that consider the mathematization of nature as a central aspect of the Scientific Revolution. Then we discuss the resignification of the thesis by Thomas S. Kuhn and Richard Westfall, who believed that the mathematization of nature has to be taken as equally relevant as other aspects in the historiography of the Scientific Revolution.[5] We conclude presenting critical perspectives to the mathematization thesis and its capacity to synthesize scientific development.

Edmund Husserl: The Idealization of Nature

Edmund Husserl (1859–1938) is one of the authors responsible for the elaboration of the narrative of the science birth as a rupture with the qualitative view adopted by the Scholastic philosophers. Husserl, in his book *The Crisis of European Science and Transcendental Phenomenology* (1936), explores the mathematization of science, aiming at understanding its philosophical meaning and assumptions. The main goal of his work is to understand what he considers a crisis in the foundations of science at the beginning of the 20th century (Carr 1970, xvi), linked to a broader crisis of the European civilization and philosophy. Such a crisis was characterized by the demand that science ought to be based on rigorously objective grounds, which reduced the scope of legitimate research questions (Husserl 1970, 5-7). The material prosperity produced by the so-called positive sciences impelled humans to neglect issues of value as the meaning of life, which according to Husserl are decisive to genuine humanity: "Merely fact-minded sciences make merely fact-minded people" (Husserl 1970, 6).

Husserl adopts a "historic-teleological" analysis to seek in the past the roots that lead to the crisis (Husserl 1970, 3). Taking Galileo as an example, he philosophically reconstructs the process of the mathematization of nature. Husserl claims that the mathematization of nature was done through the process of idealization,[6] understood as a landmark of modern science. The author considers it a milestone because it represents a rupture with the old way of grounding knowledge about nature in the accumulation of immediate observations (Husserl 1970, 23). According to Garrison (1986, 330-1), Husserl's idealization occurs in two stages: idealization$_1$ and idealization$_2$. The first is an ascending movement from the material world of sensations to the world of abstractions. Idealization$_1$ gives rise to the conceptualization of the abstract geometric objects. In the world of our perceptions, we have access to "proto-geometric" objects with irregular shapes that we perceive in a hazy way due to the imprecision of our senses. With successive measures, we are capable of softening those defects, arriving at more regular shapes. Approximate objects are enough for practical purposes, such as land-surveying and architecture. Nevertheless, for analytical purposes, it is necessary to construct perfect geometrical objects by extrapolation of the original series of measurements, arriving at ideal objects. The process of idealization$_2$ follows the inverse path. While the first departs from the world of perceptions to construct ideal objects, the second, that assumes the first, uses abstract objects as 'guides' to inquire about the world of perception, and substitute our diffuse judgments of the objects by more precise

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[5] John Henry (2002) is another important historian of science that considers the mathematization of nature as central in the development of science.

[6] From more on the process of idealization in Husserl, see Garrison (1986, 333-5).

ones.[7] Husserl considers Galileo as a Platonic-Pythagorean thinker[8] (Gandt 2004, 70-74; Palmerino 2016, 30), and criticizes him for believing that geometry could be applied to nature without further considerations. To the German philosopher, Galileo failed to reflect on "how the free, imaginative variation of this world and its shapes results only in possible empirically intuitable shapes and not in exact shapes […]" (Husserl 1970, 49). To Husserl, even when we direct our attention to the shapes of the objects, we do not experience an ideal geometric body, but one with the "effective content of the experience". Even if we transform the sensible shapes by the imagination, we always obtain other sensible forms that can only be thought in terms of gradations of straightness, flatness, circularity, but not in absolute terms of perfect geometric shapes[9] (Husserl 1970, 25). In short, Husserl blames Galileo for doing a "surreptitious substitution of the mathematically substructed world of idealities for the only real world, the one that is actually given through perception, that is ever experienced and experienceable" (Husserl 1970, 48-49). Husserl's goal is not to make a historical interpretation, but a reconstruction to serve a philosophical reflection aiming the comprehension of the meaning of mathematization in the context of the "new science". He intends to establish the "unavoidable necessity of a transcendental-phenomenological reorientation of philosophy" (Husserl 1970, 3). He focuses his analysis on Galileo because, in his understanding, it is in the Galilean physics that the mathematization of natural entities appears entirely developed for the first time.[10] However, Husserl points out that it would be necessary a more careful historical analysis to ascertain the dependencies of Galileo on his predecessors (Husserl 1970, 57).[11] Still, Husserl's interpretation brings to the foreground one of the key points explored in this paper, which is the implicit substitution of the objects of the world by mathematical idealities. As the philosopher himself points out, the substitution seems so obvious and trivial to scientific practice that it is rarely problematized, contextualized, or justified (Husserl 1970, 24). Besides that, Edmund Husserl occupies a relevant historical place in the development of the history of the mathematization of science due to his influence over prominent historians, such as Alexandre Koyré.

The Mathematization Thesis

Alexandre Koyré:
From a Heterogeneous to a Homogeneous Space

Alexandre Koyré went to Göttingen, Germany, to study in 1908. There he took classes with Hermann Minkowski (1864–1909), David Hilbert (1862–1943), and Edmund Husserl, initially studying fundamental problems of mathematics. His conception of mathematics is based on a mathematical realism, of a Platonic and Cartesian type, being deeply influenced by Husserl (Gandt 2004, 97-104; Condé 2017, 35). This kind of mathematical realism had a significant

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[7] This procedure comprises only mathematically describable aspects, as shape and position, excluding subjective ones as color and smell. For Soffer (1990, 68), the distinction between primary and secondary qualities is the cornerstone of the ontology of the new science in Husserl's reconstruction.
[8] Husserl is not the first one to characterize Galileo as Platonic. This is a thesis advocated at least since 1882 by Paul Natorp in *Galileo as Philosopher*. For a history of the thesis, see Matteoli (2019).
[9] Husserl's stance on this matter can be criticized by pointing out that Galileo considered the imperfection of real bodies and phenomena was not a hindrance to the thesis that the world is essentially mathematical, given that mathematical entities can be equally complex and imperfect (Palmerino 2016, 39-41).
[10] See Durt (2012) for a discussion on Husserl's genealogy of the mathematization of nature.
[11] Gandt (2004, 72) points out that Husserl erroneously attributes to Galileo contributions by Kepler, Descartes, Huygens, Boyle and Newton.

influence over his reading of Galileo and his historiography (Gandt 2004, 97-104). For instance, this influence can be noted when Koyré claims that modern science excludes the study of humans from its scope by replacing the qualitative world of sensory perceptions by an objective and quantitative one, a typically Husserlian thesis (Gandt 2004, 99). Husserl's influences[12] can also be noted on the fact that he defended that the mathematization of nature is not just a fundamental point in the establishment of modern science; it is also an epistemological attitude distinct from the Aristotelian philosophy.

Therefore, in the context of the present paper, we ask what is the place of the mathematization of nature, so dear to Husserl, in Koyré's historiography? An important one, but not the central, being subordinated to notions such as inertia and Galileo's concept of motion and inserted in the transition from a finite and orderly cosmos to an infinite universe and homogeneous space.[13]

For Koyré, the identification of space with geometrical space is crucial for the development of the concept of inertia. In the Aristotelian physics, space is not homogeneous, having particular points and zones (such as natural place and sublunary and superlunary zones), and motion is a particular phenomenon in the broader category of change (that includes, for instance, the growth of trees). In contrast, Galileo (still according to Koyré) considers motion as a translation from one point of a homogeneous and geometrical space to another. Besides the body's position, nothing more changes while it is moving, which implies that its state can only be recognized if compared to other bodies. As a consequence, motion is considered ontologically identical to rest. That makes the Aristotelian notion of distinct points in space superfluous, once motion ceases to be a transitory phenomenon with a goal and becomes a state (Koyré 1943a, 336-9).

For the French-Russian historian, thought experiments are part of mathematical reasoning and test the very consistencies of theories. He emphasizes the subordination of experiments to an *a priori* mathematical reasoning on Galilean physics. Natural laws are previously established by logical and mathematical deduction. In Koyré's interpretation, Galileo would only perform experiments once mathematics had already established the conclusion:

> Thus *necesse* determines *esse*.[14] Good physics is made *a priori*. Theory precedes fact. Experience is useless because before any experience we are already in possession of the knowledge we are seeking for. Fundamental laws of motion (and of rest), laws that determine the spatio-temporal behavior of material bodies, are laws of a mathematical nature. Of the same nature as those which govern relations and laws of figures and numbers. We find and discover them not in Nature, but in ourselves, in our mind, in our memory, as Plato long ago has taught us. (Koyré 1943a, 347)

In the well-known passage of the *Dialogue Concerning the Two Chief World Systems* about the relativity of motion, Galileo says that when one releases a rock from the top of a mast of a ship in movement, the stone will fall in a rectilinear trajectory, hitting the mast's base. Simplicio asks Salviati whether he had performed the experiment to reach this conclusion. Salviati answers, "without experiment, I am sure that the effect will happen as I tell you, because it must happen that way" (Galilei 1967, 145).

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[12] The influences of Husserl on Koyré are an object of debate in the literature. See Schuhmann (1987) for a critic of this influence and an exposition of the relationship between the two; and Parker for a defense of this influence. Parker (2017, 246-247, 266-247) also claims that Koyré's work should be seen as a contribution to phenomenalism.

[13] For more on this topic, see Koyré (1943a), and Cohen (1994, 75).

[14] Respectively "necessity" and "existence" in Latin.

Koyré mentions that in the 16th century, it was commonplace to consider that the main issue distinguishing Aristotle and Plato was the role attributed to mathematics on the study of nature (Koyré 1943b, 420-1). Koyré quotes the explicit discussion in Galileo's dialogues about mathematical studies of nature. According to Koyré, Sagredo had advocated a Platonic position and convinced the Aristotelian Simplicio to change his mind. Koyré portrays Galileo's motivation on the inquiry about the free fall of bodies in the dispute between Aristotle and Plato. By arguing that motion is also subject to mathematical representation, Galileo dismisses the Aristotelian position that nature does not conform to mathematical precision (Koyré 1943b).

In his historiography, Koyré attributes a central role to the mathematization of 17th century science, with particular emphasis on the mathematical realism, arguing for the priority of theory over the experiment. Koyré still proposes a 'root' to the new mathematical thinking, which would be the Platonic idealism, contrasting with the Aristotelian thought of the scholastics. In constructing his narrative, Koyré uses Husserl's ideas, by emphasizing the shifting to an *a priori* mathematical thought, and by putting Galileo on the focus of the narrative. However, while Husserl practically identifies the rise of modern science with Galileo's work, Koyré adds nuances to the story.[15]

Nevertheless, Koyré is not the only historian to place mathematics in the center of the Scientific Revolution. Eduard Jan Dijksterhuis (1892-1965) and Edwin Arthur Burtt (1892–1989): share the mathematization thesis with Koyré, but with distinct focus and interpretations.

Eduard Jan Dijksterhuis:
Mathematics in the Mechanization of the Worldview and the Hypothetico-Deductive Method

Eduard Jan Dijksterhuis dedicated a substantial part of his professional life to teaching mathematics and sciences in secondary school. Only on a late stage of his life, in 1953, he assumed a recently created chair on the history of science at Utrecht University, Netherlands. In his studies as a historian, Dijksterhuis shared Koyré's views about the centrality of mathematics to the development of modern science. However, their views have differences that illustrate the nuances of the mathematization thesis. Dijksterhuis synthesizes his perspective on the topic in the paper *Designed for Grasping Quantities* (1955).[16] He considers that the fecundity of the relation between mathematics and physics should cause "wonder" because it is not evident that mathematics, a free creation of the human mind, is related to science, which deals with a reality indifferent to the human action (Dijksterhuis 1990, 115). According to him, if we feel indifferent to this, it is because our scholar education naturalized this relation. For Dijksterhuis, the comprehension of the relations between mathematics and physics through history is constrained to the understanding of the mechanistic view of the world[17] and the construction of the hypothetico-deductive method (Dijksterhuis 1990, 118).

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[15] Koyré mentions, for instance, other thinkers that tried to solve the Aristotelian objections to the Earth's movement. He also points out that Galileo never assumed a definitive position concerning the infinity of the universe, restraining himself to deny its limitation by a physical sphere of fixed stars (Koyré 2006, 88). See also Parker (2017, 267,270) for distinctions between Koyré and Husserl.

[16] Originally published in Dutch under the Latin title *Ad quanta intelligenda condita* in reference to Kepler's expression. Here we use H. F. Cohen's English translation of 1990.

[17] See Feldhay (1994) for a criticism of the bias that the commitment with this theme imposes on Dijksterhuis' narrative.

In the book *The Mechanization of the World Picture* (1961),[18] the Dutch author adopts a historical perspective to analyze the emergence of mechanicism; the roles that mathematics plays in modern science, and what differentiates the ancient applied mathematics from, for example, Galileo's and Newton's mathematical mechanics.

Dijksterhuis considers the mechanization of the world-picture as responsible for the most significant and comprehensive changes in science and society. He believes that the adequate definition of said view is the one that demands an explanation of the phenomena in Newtonian terms, which are different from 'mechanic' as understood in analogy to machines. A mathematized mechanics does not mean the mere use of mathematical expressions and words, which could be replaced by a common language, but the ontological sense of mechanical concepts and laws being mathematical (Dijksterhuis 1961, 499). Therefore, the centrality of the mathematization is hidden by the use of the term *mechanization* in the book's title. Cohen claims that *The Mathematization of the World-Picture* would be a more appropriate title for Dijksterhuis' book (Cohen 2016, 146).

Dijksterhuis' narrative is organized around the development and implementation of the hypothetico-deductive method, highlighting the complementarity and hierarchy of mathematics and experimentation. Experiments are designed after the formulation of mathematical hypotheses, deductions, and predictions (Dijksterhuis 1961, 70-1). The roles of mathematics in what he calls hypothetico-deductive method are to describe a set of empirical data, express a hypothesis that explains the data, and to work as a tool for deducting new possible observations (Dijksterhuis 1990, 117). Dijksterhuis uses the method as a historiographical category to analyze the development of scientific practice, pointing out when certain stages are or are not followed.

For instance,[19] Dijksterhuis highlights the mathematical character of *On the Equilibrium of Planes* by Archimedes. The starting point of its demonstrative structure are axioms considered self-evident or observable by analogy with what we see through our sense, followed by mathematical idealizations of material objects and mathematical deductions with physical conclusions. Confirmatory experiments are unnecessary, i.e., the last stage of the hypothetico-deductive method is not fulfilled (Dijksterhuis 1990, 115-118). Ancient astronomy is a distinct case because the movement of celestial bodies cannot be noticed exclusively by sensory perceptions. Therefore, the starting point of inquiry must take into account sets of quantitative data besides the axioms. The formulation of the hypothesis is constrained by "*a priori* assumptions which rested on religious, aesthetic, or scientific grounds" (Dijksterhuis 1990, 116). As an example, Dijksterhuis mentions the "platonic axiom", which states that the movement of celestial bodies is circular and uniform. Ptolemy had used this axiom to describe the Sun's movements and to predict its future positions (Dijksterhuis 1990, 116).

It is crucial to notice that Dijksterhuis do not consider the use of the hypothetico-deductive method as the only relevant feature to explain scientific development. Dijksterhuis views scientific results acquired until the 16th century as "tiny", even despite the contributions of, for instance, Claudio Galen (129-217) and Robert Grosseteste (1168–1252) (Dijksterhuis 1990, 118).[20] For Dijksterhuis, the development of mechanics (that he considered the prototype of classical physics) advanced only when thinkers, influenced by Archimedes,

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[18] Originally published in Dutch in 1950 under the title *Mechanisering van het wereldbeeld*. Here we use C. Dikshoorn 1961 translation.

[19] For a more detailed analysis of the mentioned examples, see (Dijksterhuis 1961, 1990).

[20] He considers this 'delay' on the capacity of the method to generate results as an open issue in history, whose analysis hinges on several factors. Among them, he mentions the underestimation of the difficulties of natural inquiry by the Greeks (who used hypothesis without proper empirical confirmation), the qualitative character of the broadly accepted Aristotelian physics and the difficulty of the creation of an experimental method, among others (Dijksterhuis 1990, 119).

disengaged themselves from Aristotelian orthodoxy. For instance, he finds Simon Stevin's (1548–1620) research on statics and hydrostatic as connected with Archimedes' theories (Dijksterhuis 1990, 120).

Dijksterhuis takes Galileo as a central figure in his historiography. For him, "with perfect acuity, Galileo establishes once and for all the scientific method for the study of inorganic matter" (Dijksterhuis 1961, 339). He takes Galileo's work on free fall as an epitome of the mechanization of the world-picture and the establishment of the hypothetico-deductive method. While Aristotelian physics is concerned with the causes of phenomena, Galileo was concerned with description. He searched for motions that could describe the natural events satisfactorily, relegating the search for the causes to other philosophers. The particular concern to the proper description of the phenomena shows the hypothetical character of Galileo's work. Dijksterhuis also points out that experimentation in the Italian philosopher's work always had a confirmatory function, as suggested by the hypothetico-deductive method.[21]

About the Galilean theory of motion, Dijksterhuis states:

> Once theoretical mechanics had inferred its axioms from the study of the natural phenomena of rest and motion […], it turned from physical science to mathematics. Radically idealizing the phenomena by the elimination of all disturbing influences and schematizing everything with equal thoroughness by means of simplifying abstraction […], mechanics developed into an autonomous science, quite remote from physical reality. (Dijksterhuis 1961, 346)

This is a crucial point on Dijksterhuis' narrative, which considers the detachment of mechanics from natural phenomena a critical step in the development of the mechanical (mathematical) world picture.

According to Dijksterhuis, Isaac Newton's (1643–1727) studies on gravity contributed to improving the hypothetico-deductive method. The mathematical laws that accurately describe the free fall and the orbits of the planets follow from the same hypothesis (Dijksterhuis 1990, 123-124). Due to the success of Newton's theory and the difficulty of explaining the action at a distance in corpuscular terms, he abolished the requirement that hypotheses should "lend themselves in any case to a visual picture" (Dijksterhuis 1990, 123), being enough that they are useful to deduct the most significant amount of observable phenomena. Dijksterhuis concludes:

> Thus the expansion of the mathematical-empirical method[22] accomplished by Newton was of a principal nature and significance in that the aim of a scientific theory could now be defined as providing a mathematical description of the course of a natural phenomenon, under the naturally added clause that the description must give rise to empirically verifiable consequences. (Dijksterhuis 1990, 124)

With the development of mathematical sciences, the relation between mathematics and nature resurges, according to Dijksterhuis, as a continuity of the medieval problem of universals, which questioned the connection between idea and physical reality. The Platonic stance of *universalia ante rem* [universals before objects] is now understood as considering the world as an imperfect realization of mathematical ideals from the world of thought. The

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[21] Nevertheless, Dijkstehuis recognizes that some confirmatory experiments for Galileo's previsions were realized posteriorly (Dijksterhuis 1990, 120). Cohen (1994, 71-72) criticizes this point on Dijksterhuis' argument, because historical data shows that Galileo made more heuristic experiments than Dijksterhuis credited him.

[22] Other term for hypothetico-deductive method used by Dijksterhuis.

Aristotelian *universalia in re* (universals in objects) conceives mathematical idealization as an abstraction from physical reality. Finally, the nominalist view, *universalia post rem* (universals after objects), understands mathematics as a useful tool to obtain approximate knowledge of reality.

Edwin Arthur Burtt:
Metaphysical Consequences of Mathematization

Historical studies on the mathematization of science gained an air of robustness and strength in the 1920s, after the publication of Dijksterhuis and Edwin Arthur Burtt books, both in 1924. In this year, Burtt presented his Ph.D. thesis, *The Metaphysical Foundations of Modern Physical Science: A Historical and Critical Essay*,[23] at Columbia University, the center of American pragmatism. Burtt had a significant influence on his coetaneous, for instance, Koyré changed the focus of his studies from the history of religion to history of science after reading Burtt's book[24] (Villemaire 2002, 3-4, 49).

According to Lorraine Daston (1991, 523-4) and Diane Villemaire (2002, 49), Burtt uses the term 'metaphysics' denoting the assumptions that ground a scientist's work and usually are not explicit nor analyzed critically.[25] The *Metaphysical Foundations* is shaped by the conflict with the positivists[26] whose main idea is that "it is possible to acquire truths about things without presupposing any theory of their ultimate nature [...]" (Burtt 1954, 227).

The Metaphysical Foundations can also be taken as a seed of the approach later advocated by Kuhn (1977) and Westfall (1971), who consider the existence of two traditions in the historiography of the Scientific Revolution. Burtt recognizes the presence of a parallel current of studies on natural philosophy exemplified by William Gilbert (1544–1603), William Harvey (1578–1657), and Robert Boyle (1627–1691). Their empiricist approach contrasts with a kind of mathematical reductionism, exemplified by Galileo.

Burtt (1954, 329) often reiterates the inevitability of the metaphysics because while trying to avoid it, we just end up adopting a hidden, uncritical metaphysics. He directs his attention to what he considers the main problem of modern philosophy – whether we can reach trustful knowledge, and how we do it. For the medieval philosophers, humans not only occupy the central physical place in the world, but also the world was made for us and, therefore, our ability to understand it was accepted *a priori*. Burtt's goal is to enable the development of a new metaphysics that is compatible with modern science and simultaneously ensures a central place for humans (Burtt 1954, 304; Villemaire 2002, 52).

According to Burtt, modern philosophers failed to accomplish this task. Burtt speculates that this failure is due to an uncritical adoption of the categories of modern science that deeply contrasts with the old ones. It does not mean that modern philosophers blindly accepted Newton's categories, but none of them has subjected the whole of them to critical reflection (Burtt 1954, 35). The categories of substance, accident, causality, essence and idea, matter and form, potentiality and actuality are replaced by force, motion, laws, changes of mass in space and time. Therefore, in order to understand how this drastic change happened, it was necessary to investigate the origins of philosophical thought between 1500 and 1700. Burtt's investigation is distinct from Koyré and Dijksterhuis', who were interested

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[23] See Moriarty (1994) for an interpretation of Burtt's works as a whole, and Chatzigeorgiou (2020) for a survey of the interpretations of Burtt's work.

[24] Among other historians influenced by Burtt are Thomas Kuhn and Robin George Collingwood (1889-1943).

[25] Definition posteriorly criticized by the historian Lorraine Daston (1991).

[26] 'Positivists' can denote a diverse group of thinkers but, according to Daston, Burtt refers to the ones from the 1890s, exemplified by Ernest Mach.

in constructing a historical overview of scientific ideas in the 17th century, while Burtt is involved with its philosophical consequences (Cohen 1994, 92). For him, the modern worldview structured by Isaac Newton influenced philosophers and the learned audience in general for being responsible for the creation of a new set of knowledge for the European intellectuality "such that all problems must have been viewed afresh because they were seen against it" (Burtt 1954 32).[27]

Therefore, *The Metaphysical Foundations* aims at a historical analysis of the origins of Newton's philosophical assumptions. Burtt begins with the question: why Copernicus and Kepler did compromise themselves with the defense of the heliocentric model against our sensory perception of the stillness of the Earth and a well-established natural philosophy? According to him, the answer lies in mathematics. For Copernicus and Kepler, the heliocentric model was more straightforward and more harmonic than the geocentric. Simpler because it reduced the number of necessary epicycles to explain the phenomena from eighty to thirty-four;[28] more harmonic because it represented the movements of all celestial bodies as concentric circles around the Sun, except the Moon (Burtt 1954, 36-39). Nonetheless, the question of why they prioritized the mathematical arguments over the philosophical and empirical ones remains.

He discusses the rise of the Pythagorean and Platonic thought[29] that asserts the mathematical nature of reality (Burtt 1954, 40-41, 207-209). Burtt considers Kepler's understanding of cause as a mathematical harmony between diverse phenomena. It is a mathematical reformulation of the Aristotelian formal cause. Kepler appropriated the notion of primary and secondary categories from the ancient atomists and skeptics who states that the world presented to our senses is not an ultimate reality, but an expression of it (Burtt 1954, 64).

Burtt also puts Galileo in the spotlight, whose mathematical approach of movement is encouraged by the Copernican model. In this model, there is homogeneity between Earth and the skies, suggesting the possibility of a mathematical approach to earthly phenomena, given the validity of mathematical methods of movement of celestial bodies. It makes Galileo, according to Burtt, the first to consider the possibility of a complete and precise description of the movement of earthly bodies (Burtt 1954, 112).

Burtt's interest is to understand the metaphysical implications of the mathematical approach in the study of free fall. One of them is the change of emphasis on the question of *why* the bodies fall to *how* they fall. In his studies of free fall, Galileo used different categories than Aristotle had used. They were not explicitly formulated but can be inferred from how Galileo used them in his studies. Mainly, the notions of space and time acquired new meanings in the new metaphysics (Burtt 1954, 91-8).

Burtt, as the previously mentioned authors, explores the Galilean mathematical apriorism, showing how some parts of Galileo's writings may suggest extreme interpretations. Among them, Galileo stated that experiments are a way to convince, not a necessary part of the new science. However, Burtt gradually dilutes the boldness of this kind of statement. He shows that they do not express a complete disconnection of Galileo from experience and observation; according to Burtt, Galileo meant that it is possible, with few

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[27] Burtt saw Newton as an outstanding scientist but, as philosopher "he was uncritical, sketchy, inconsistent, even second-rate" (Burtt 1954, 208). Still, the Newtonian metaphysics ended up being implicitly accept because of its success (Burtt 1954, 230).

[28] Burtt is inaccurate in this point. Ptolemy's system, after Georg von Peuerbach's (1423–1461) reformulation, used forty circles, while the most sophisticated version of Copernicus' system in the *De Revolutionibus* used forty eight (Martins 2003, 82).

[29] Daston (1991) criticizes Burtt's emphasis on this point, arguing that it is not a satisfactory explanation because not all thinkers explored in Burtt's work as responsible for the ascension of the mathematical view of the world were Platonic.

experiments, to extract the mathematical essence of phenomena. Following such essence, it is possible to deduce valid conclusions to all similar instances, even when they are not subject to observation. Burtt summarizes Galileo's method in three stages: intuition or resolution starting with phenomena to extract an underlying mathematical structure; demonstration making generalizations and deductions of new properties following the intuited structure from the previous step; and experimentation to verify the deductions (Burtt 1954, 81).

According to Burtt, the core of the metaphysics of modern science is the "ascription of ultimate reality and causal efficacy to the world of mathematics, which is identified with the realm of material bodies moving in space and time" (Burtt 1954, 303). The explanatory categories of science are defined according to the possibility of its mathematical expression. He also notes that over the Scientific Revolution, natural philosophers started to avoid metaphysical issues, except when they could be used to legitimize the mathematical conquest of the world (Burtt 1954, 306). Thus, for Burtt, as for Koyré and Dijksterhuis, the mathematization of the physical world plays a central role in the narrative about the Scientific Revolution.

Cohen (1994, 88-89) and Villemaire (2002, 187) advocate that the relevance of *The Metaphysical Foundations* was not fully recognized, and both authors consider the book as the first contextual historical approach to the development of science in a discontinuous way. That may be due to the hostile context of the time when the book was published. It was a usual practice of the American philosophers of the 1930s to keep surveillance over philosophical groups that acknowledged the possibility of attaining trustful knowledge beyond logical and empirically. The *Metaphysical Foundations* was considered as a door to a less formal style of thinking. Another factor that illustrates the hostility to Burtt's approach to the history of physics was the significant influence of the historian George Sarton (1884–1956), founder of the *History of Science Society,* who considered the philosophy of science as useless to the advance of scientific knowledge (Villemaire 2002, 190). It is noteworthy that Burtt had prepared the grounds for the Kuhnian historiography that broke with Sarton's tradition, and the next period would be more friendly to Burtt's style of historiography.

Synthesizing the Mathematization Thesis as a Historiographical category

The limits of the mathematization thesis are not clear due to its malleability as a historiographical category. They will become more evident in the next sections where we expose the dilution and criticism of the program. Here we synthesize the main differences among the three historical works analyzed to illustrate the flexibility of the thesis.

Koyré and Dijksterhuis contrast in terms of the main narrative of their works. For Koyré, mathematization allows the substitution of the conception of a limited cosmos by an infinite universe. Dijksterhuis considers the process of mathematization as tightly entangled to the origin of a mechanical world picture, identifiable with the birth of science. However, the most critical difference between both authors lies in their views on the epistemic roles of mathematics. Koyré assumes a Galilean vision of mathematics as the language of nature. A result of this view is that the elaboration of a mathematical theory about natural phenomena can lead to, at least potentially, unveiling its reality. In contrast, Dijksterhuis considers mathematics as descriptive since it establishes quantitative relations between different entities, without mentioning their essences. For Dijksterhuis, mathematization corresponds to one stage of the scientific method that should be complemented by experiments, while Koyré considers experiments less relevant.

Burtt's approach diverges from Koyré and Dijksterhuis' in its primary goal. While Koyré and Dijksterhuis intend to comprehend the birth of modern science, Burtt focuses on the

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underlying philosophy that substantiates modern science and its ulterior impact on humans' self-image.

Since the three historians are distinguished for their contribution to the establishment of the mathematization thesis as central to the Scientific Revolution, one can ask why Koyré is the most famous name? Cohen (2016) points out some reasons. He first considers Dijksterhuis' linguistic isolation, whose first book, *Val en Worp*, published in the same year as Burtt's *Metaphysical Foundations* (1924), was written in Dutch. Dijksterhuis' *The Mechanization of the World Picture* was also initially published in Dutch, being translated into English only in 1961, an inadequate translation, according to Cohen (2016, 151). Besides, Dijksterhuis worked mainly as a teacher and oriented his public lectures in the Netherlands mostly to a lay audience. Burtt also was not very committed to promoting his *agenda* due to his modest personality. On the other hand, Koyré's success in propagating his thesis is a combination of his character with a cosmopolitan personality. Over his career, he transited between Germany, Egypt, France, and the United States and was also more devoted to advocating the mathematization thesis in the context of the Scientific Revolution (Cohen 2016, 150-152).

The mathematization thesis attracted attention due to its efficiency in establishing a cohesive narrative that brought together prominent characters such as Galileo, Descartes, Kepler, and Newton in a coherent narrative. At the same time that the focus on mathematization made the narrative attractive, it also made it fragile. Since the 1950s, its status started to be 'diluted' by Herbert Butterfield (1900–1979), Marie Boas Hall (1919–2009), and Alfred Rupert Hall (1920–1979). The first extends the Scientific Revolution's scope to the period between 1300 and 1800, including the *impetus* dynamic and the late revolution in chemistry consummated by Lavoisier, thus comprising periods and events in which mathematics could not be seen as central. Nevertheless, Butterfield still considers the existence of a 'central' Scientific Revolution (similar to the Scientific Revolution described in the mathematization thesis) inside this broader period. Marie Hall and Alfred R. Hall characterize the origin of modern science by the prevalence of rational conceptions and methods, contrasted with those of magic, mysticism, and superstitions, with mathematics being one element among others that contributed to the birth of the new science (Cohen 2016).

It is no wonder that the mathematization thesis was subjected to criticism. After all, it minimizes the contributions of empirical natural philosophers such as Bacon, Harvey, Boyle, and Hooke to the Scientific Revolution; it neglects fields of knowledge that were mathematized later, as medicine and chemistry; without saying that it disregards the social context in which the birth of the new science took place. In the 1970s, Thomas Kuhn proposed a historiographical approach that considers a broader range of events as relevant without neglecting the developments attained by the historians of the mathematization thesis.

Thomas Kuhn and the Two Traditions

Thomas Kuhn, in the 1976 paper "Mathematical versus Experimental Traditions in the Development of Physical Science", presents an alternative interpretation that copes with some of the weaknesses of the mathematization thesis without sacrificing its potential of syntheses. He starts with the question: is science a single cohesive body of knowledge with a shared history, or is it an assembly of independent disciplines with distinct histories? (Kuhn 1977, 31).

The approach, which considers mathematics, physics, astronomy, chemistry, biology, and geology a cohesive body of knowledge called science, usually focuses on the intellectual, ideological, and institutional context in which science flourished, neglecting the conceptual

evolution of the individual disciplines. However, Kuhn considers that the development of institutions, values, methods, and scientific worldviews do not exhaust the study of scientific development (Kuhn 1977, 33). Those who consider science as a collection of independent disciplines are immune to this criticism due to their focus on technical content. Nevertheless, they can be criticized for reconstructing the limits between these disciplines in an anachronistic manner.

Recognizing at same time science's unity and disciplinary particularities, Kuhn proposes the existence of two parallel traditions, the classical mathematical sciences and the Baconian sciences. They differ in institutional structure, interaction between their disciplines, tendencies of practitioners of one subject to contribute to another, and shared intellectual and material prerequisites for their practices.

The classical sciences include astronomy, statics (including hydrostatic), optics, mathematics, and harmony. According to Kuhn, it is the only field of knowledge that can eventually be recognized as physical sciences that progressed unequivocally since Antiquity, considering the accumulation of concrete and permanent knowledge as a measure of progress. The study of movement around the 14th century can be included in the classical sciences due to the Scholastic developments that detached it from the problem of broad qualitative changes, as it was conceptualized in the Aristotelian philosophy. The two main features that unite the six disciplines are their predominantly mathematical nature and relatively *a priori* character.[30]

Kuhn considers the Baconian sciences in contrast to the experimental practices of the ancients. According to him, experiments used to be performed to reinforce a previously known result or to offer an answer to a question posed by an already existing physical theory. Besides, several ancient experiments were mental, being difficult for the historian to establish the ones that were performed and those that were not. In contrast, the experiments made by Baconians such as Gilbert, Boyle, and Hooke aimed to investigate and discover and were more valued than theory. They seek to verify how nature behaves in previously unknown circumstances, even under artificial conditions that existed only in laboratories-experiments that Bacon described as "twisting the lion's tail". (Kuhn 1977, 43-44).

Some of the Baconians were influenced by atomist or corpuscular theories and metaphysical conceptions. However, the gap between theory and experiment was deep. Therefore, the primary goal of experiments was to make an inventory of empirical facts that eventually could ground a coherent body of theoretical work. In addition, Baconian experimentation was characterized by the use of sophisticated instruments, as barometers, telescopes, microscopes, air pumps, thermometers, among others.

In this scenario, one can ask whether classical mathematical sciences were influenced by Baconian experiments. Kuhn answers no, at least on the conceptual level. Even when experiments were performed, they usually confirmed previously known results, being sophisticated versions of ancient ones or the embodiment of old questions. Even when experiments revealed new phenomena related to the disciplines of classical sciences, it took a long time until they were incorporated into theory; for instance, in the case of optics, experiments of polarization, diffraction, and interference (Kuhn 1977, 45-6).

Kuhn justifies the separation between disciplinary traditions based, among other factors, on Bacon's attitude towards mathematics. Bacon considered mathematics and its 'quasi deductive' structure of classical sciences as untrustworthy. In bringing attention to the importance of the experimental tradition, parallel to the classical sciences, Kuhn dilutes the

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[30] Despite the fact that ancient mathematical science acknowledged empirical observations, they are considered *a priori* because they were independent of more complex and experiment-dependent observations. Following simple observations, theories could be elaborated deductively using mathematical concepts (Kuhn 1977, 35-6).

role of mathematics relative to the supporters of the mathematization thesis, while maintaining a protagonism to the mathematical current:

> If, therefore, one thinks of the Scientific Revolution as a revolution of *ideas*, it is the changes in these traditional, quasi-mathematical fields which one must seek to understand. Although other vitally important things also happened to the sciences during the sixteenth and seventeenth centuries (the Scientific Revolution was not merely a revolution in thought), they prove to be of a different and, to some extent, independent sort. (Kuhn 1977, 41)

This point is reinforced by Kuhn's statement about the underdevelopment of the Baconian sciences in the 17th and 18th centuries. He considers them as underdeveloped because they were unable to produce consistent theories that gave rise to accurate predictions (Kuhn 1977, 47). Consequently, Kuhn agrees with Koyré when he considers the theoretical reconceptualization of motion as more relevant than the development of experimentation:

> After all due qualification, some of it badly needed, Alexandre Koyré and Herbert Butterfield will prove to have been right. The transformation of the classical sciences during the Scientific Revolution is more accurately ascribed to new ways of looking at old phenomena than to a series of unanticipated experimental discoveries. (Kuhn 1977, 46)

In the book, *The Construction of Modern Science: Mechanism and Mechanics* (1971), Richard Westfall (1924–1996) presents a similar interpretation but using different categories and assuming their interaction. Adopting Koyré's view that Platonic and Democritian currents were the constitutive elements of the new science, Westfall considers that they interacted in the 17th century in a quarrelsome manner. Despite the rejection of the qualitative descriptions of nature by the mechanical philosophy, the demand for a mechanical explanation for the phenomena was an obstacle to the complete mathematization of nature (Cohen 1994, 138-9). For Westfall, the culmination of the Scientific Revolution happens with Newton's work that unified both currents.

The approach of the two currents did not have continuity due to several factors. Among them, the progressive establishment of social studies of science causing a departure from the history of ideas towards sociological historiographical approaches; the growth on the number of professional historians; the decrease of scientists in the practice of the history of science; and the origin of continuist views that linked the 17th thinkers' conceptions with philosophers from the Middle Ages, minimizing the revolutionary character of science and, hence, of the mathematization as a division criteria between the ancient and the modern sciences.

Critics of the Mathematization Thesis

After the dilution of the relevance of the mathematization thesis in the historiography, new critics to it were weaved. The philosopher Gary Hatfield (1951 -) criticizes the philosophical roots of mathematization exposed by Koyré and Burtt. He questions the existence of a Platonic-Pythagorean metaphysical basis shared by the main actors of the Scientific Revolution, claiming that their philosophical bases were different, having only

> a broadly shared attitude towards the relationship between mathematics and nature amounts to a simple statement that mathematical modes of description are useful in

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the investigation of nature, a proposition that hardly qualifies as Pythagorean and Neoplatonic or as Metaphysical. (Hatfield 1990, 94)

Hatfield sustains that Koyré and Burtt's interpretation blurs Galileo's philosophical contribution, which was to understand "how one can seek to establish the appropriateness of one type of approach to natural science over its competitors without first establishing a metaphysical framework as foundation and support" (Hatfield 1990, 118).

Despite the possibility of identifying Platonic and Aristotelian influences in Galileo, Hatfield claims that he justified the value of the mathematical approach by its large number of well succeed examples, and not by philosophical bases. He explicitly criticizes some justifications used by Koyré to support the weight of the Platonic influence over Galileo as the doctrine of reminiscence in the *Dialogue Concerning the Two Chief World Systems*. Hatfield advocates that Salviati was ironic. He also points out that the mentions to Plato in the *Dialogue* always occurred in Simplicio's and Sagredo's lines, but never in Salviati's ones (Hatfield 1990, 120-5).

Lorraine Daston (1951 -) criticizes Burtt's analysis by pointing out that he disregarded the intellectual context that had allowed him to explain the origin of the metaphysical conceptions of the main characters. According to Daston, the setting provided by Burtt is "incidental, biographical, and pointedly nonrational". She partially attributes this negligence to a

> [...] lingering psychologism admixed with positivist prejudices: presuppositions can be accepted or rejected only on faith, and thus predispositions (private, ineffable, idiosyncratic) are paramount. In part, it is due to an implicit view of history, or at least of the history of philosophy, as having the forward momentum of a hurtling locomotive; ideas develop along certain lines because they must do so. (Daston 1991, 524).

Daston considers the relationship of necessity amid the distinction between primary and secondary qualities and the mathematization of science traced by Burtt as fragile. According to her, this is due to Burtt's underestimation of the different interpretations of the distinction between primary and secondary qualities, and to his inappropriate assumption that mechanistic philosophy implies mathematization. This makes him inattentive to works that incorporate the mechanical philosophy and the distinction between primary and secondary categories but do not implement mathematization, as Descartes' *Principia* and a substantial part of Boyle's work (Daston 1991, 526-7).

Steven Shapin (1943 -) criticizes the mathematization thesis on the sociological level. He considers mathematics as a language unable to foster the formation of the community of practitioners of experimental sciences that flourished in the 17th century. His goal is not to oppose philosophical arguments to the social value of mathematics, but to show that they are intertwined. The use of mathematics as a language to convey scientific statements was criticized for its supposed lack of intelligibility, except within a restricted community. Intelligibility, in its turn, was a precious value in the period. Boyle, for instance, considered it as a virtue that justified the adoption of the mechanical philosophy over the Scholastic. Moreover, intelligibility was necessary for the formation of a broad community capable of evaluating scientific statements, which Boyle considered a requisite for the establishment of a true physics (Shapin 1994).

Yves Gingras (1954 -) criticizes Koyré stating that the mathematization of nature was a progressive extension of the ancient mixed mathematics instead of a cultural boost caused by Neoplatonic influences. Nevertheless, the core of his critic resides in showing the collateral effects caused by the mathematization, made invisible by the habit of the historians to pay excessive attention to the 'winners.' According to him, this habit erroneously conveys

the idea that, after Newton's *Principia,* the applicability of mathematics to nature became obvious and ceased to be opposed (Gingras 2001). Gingras structures his criticism based on epistemological, ontological, and social arguments.

The epistemological changes consisted of the modification of what was understood by *explanation*. The success of the Newtonian mathematical approach to gravitation influenced an increasing number of natural philosophers to take the Newtonian mathematical description of motion as its explanation, dispensing the need for mechanical causes. The ontological consequence was that science starts to deal more with the relation of objects and less with its substance or nature. Gingras points out Maxwell's commentary in a letter to J.A. Fleming in which he states, "the progress of science was indicated by our making our terms mean less and less". He meant that physicists ceased to talk about categories as electric fluid and caloric and started to refer generically to electricity and heat, without saying what those substances are (Gingras 2001, 404). In the social dimension, Gingras points out that the mathematization of physics restricted the participants in science because knowing mathematics became an acceptance criterion for being a member of the scientific community.

Recently, Sophie Roux's (1965 -) proposed a complexification of the very notion of the mathematization of nature. She questions if the use of mathematics can make a clean cut between Aristotelian and classical quantitative physics. She points out that despite Aristotle's statement that mathematics is only capable of capturing superficial properties of the objects, there were several Aristotelian currents of thought during the Renaissance that were compatible with the introduction of mathematics in natural philosophy (Roux 2010).

She also points out the fact that the different forms of mathematizing nature varies according to the kind of mathematics used (algebra, geometry, calculus…) and according to the several facets of mathematical practice. Concerning the first distinction, she mentions how the Euclidean theory of proportions both guided and frustrated the Galilean analysis of motion, and how this analysis would be profoundly changed when transcribed to the language of calculus.

> These remarks may lead to the conclusion that the grand narrative about mathematization of nature has to be enriched with the dense spectrum of various mathematical practices. And, indeed, leaving behind the idealities that Husserl and Koyré waved at and replacing them with real practices such as manipulating numbers, extracting roots, representing perspective in pictures, compounding proportions, arranging numbers in tables, following rules and algorithmic procedures, linking propositions together, visualizing magnitudes in geometric diagrams, solving problems, measuring fields with specific instruments, drawing curves, making deductions and plotting the routes of ships, was a significant and much needed change of scenery. (Roux 2010, 328).

These various criticisms put into question the generalizations explicitly made or assumed by the mathematization thesis and its authors. We advocate that the mathematization of nature in the 17th century cannot be easily traced back to Platonic or Aristotelian philosophical roots, as indicated by Hatfiled's (1990) analysis of Galileo. Also, the extensive use of mathematics had some side-effects unrecognized by the highly idealized narrative of the mathematization thesis, such as those pointed by Shapin (1994) and Gingras (2001).

Final remarks

Historians of the science of the beginning of the 20th century considered as the founders of the mathematization thesis introduced the notion that the most notorious event on the establishment of modern science was the expansion of the mathematical approach to nature in the 17th century. In their narratives, mathematization is the main criterion that separates the medieval Scholastic philosophy from the science of the moderns, in a discontinuous rupture. For Koyré (1943a), the use of mathematics was essential in the formulation of fundamental laws of modern science, such as inertia and indispensable for the transition from a notion of a closed world towards an infinite universe. For Dijksterhuis, the use of mathematics is indicative of scientific progress in at least two ways. First, as a landmark of a new worldview, the mechanical world picture, according to which the natural phenomena are explained in mechanical terms (in the sense of a mathematical discipline) (Dijksterhuis 1961). Second, as an indicator of a supposedly methodological evolution (Dijksterhuis 1990). Burtt (1954) has a distinct goal from previously mentioned historians; his attention is predominantly devoted to the impacts of the mathematization of nature in the metaphysics of modern science, and how it entailed a diminishment of humans' position in the universe.

In the second half of the 20th century, the mathematization thesis is diluted in works such as those of Butterfield, Mary Hall, and Alfred R. Hall. They expand the period of the Scientific Revolution, including events in which mathematization had no fundamental role (Cohen 2016). In the 1970s, Kuhn (1977) and Westfall (1971) revise the relevance of mathematization in narratives of the scientific development by considering two traditions of scientific progress, mathematics being essential only to one of them.

Since the end of the 20th century and at the beginning of the 21st, the protagonism of the mathematization thesis in historical narratives has been severely criticized. According to the critics, despite the indisputable role of mathematics in contemporary science, its use in the natural sciences was not always considered natural and unproblematic. Shapin (1994) and Gingras (2001), for instance, show how natural philosophers such as Boyle and Faraday criticized the social impact of the excessive use of mathematics. Gingras also points out other consequences of mathematization on epistemological and ontological levels: the changes in the meanings of explanation and the emphasis of mathematical structures over substance.

Beyond the criticism previously presented in this paper, we can mention some narratives of the origin and development of science that take radically different principles than those of the mathematization thesis. For instance, Boris Hessen (1893–1936) relates the content of Newton's scientific work with the economic demands of its time (Hessen 2009) and Robert Merton (1910–2003) considers the fast development of the scientific activity of the 18th century as a product of puritan values.

Currently, the notion that modern science emerged from a discontinuous break with the Scholastic philosophy in the 17th century characterized predominantly by the broad application of mathematics to nature is untenable. The works of several actors as Bacon, Boyle, and Hooke do not fit this description, neither fields as medicine and chemistry. Nevertheless, if we remain aware of its exaggerations, complexities and subtleties brought up by contemporary critics, mathematization can still be a fruitful approach to writing the history of science. As an example, we mention Hendrik Floris Cohen's (1946 -) book *How Modern Science Came into the World* (2010), that extends the idea of two traditions introduced by Kuhn and Westfall to construct a historical narrative that incorporates mathematization as a critical constituent of the scientific development together with other features.

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Acknowledgments

References

Burtt, Edwin. 1954. *The Metaphysical Foundations of Modern Science*. New York: Doubleday Anchor Book.

Butterfield, Herbert. 1965. *The Origins of Modern Science 1300-1800*. New York: The Free Press.

Carr, D. 1970. Translator's Introduction. In: Husserl, Edmund. *The Crisis of European Sciences and Transcendental Phenomenology*. Evanston: Northwestern University Press.

Chatzigeorgiou, Konstantinos. 2020. How the Mind-World Problem Shaped the History of Science: A Historiographical Analysis of Edwin Arthur Burtt's *The Metaphysical Foundations of Modern Physical Science. Studies in History and Philosophy of Science*, part I. Forthcoming – https://doi.org/10.1016/j.shpsa.2020.05.002

Cohen, Hendrik. 1994. *The Scientific Revolution: A Historiographical Inquiry*. Chicago: The University of Chicago Press.

Cohen, Hendrik. 2010. *How Modern Science Came into the World: Four Civilizations, One 17th Century Breakthrough*. Amsterdam: Amsterdam University Press.

Cohen, Hendrik. 2016. The 'Mathematization of Nature': The Making of a Concept, and How it Has Fared in Later Years. In: *Historiography of Mathematics in the 19th and 20th Centuries*, edited by Volker Remmert, Martina Schneider, Sørensen Kragh, 143-160 Birkhäuser.

Cohen, Ierome Bernard. 1985. *Revolution in Science*. 1ª. ed. Cambridge, Massachussets and London: The Belknap Press of Harvard University Press.

Condé, Mauro. 2017. *"Um Papel para a História": O Problema da Historicidade da Ciência*. Editora UFPR, 2017

Daston, Lorraine. 1991. History of Science in an Elegiac Mode: E. A. Burtt's Metaphysical Foundations of Modern Physical Science Revisited. *Isis* (82): 522-531.

Dijksterhuis, Eduard. 1924. *Val en Worp: Een Bijdrage tot de Geschiedenis der Mechanica van Aristoteles tot Newton*. Groningen: Noordhoff.

Dijksterhuis, Eduard. 1961. *The Mechanization of the World-Picture*. Translated by C. Dikshoorn. [S.l.]: Oxford University Press.

Dijksterhuis, Eduard. 1990. Ad Quanta Intelligenda Condita - Designed to Grasp Quantities. *Tractatrix* (2): 111-125.

Durt, Christoph. 2012. *The Paradox of the Primary-Secondary Quality Distinction and Husserl's Genealogy of the Mathematization of Nature*. Santa Cruz: Thesis.

Feldhay, Rivka. 1994. Narrative Constraints on Historical Writing: The Case of the Scientific Revolution. *Science in Context* 7 (1): 7-24.

Feyerabend, Paul. 1993. *Against Method*. London: Verso.

Galilei, Galileu, 1967. *Dialogue Concerning the Two Chief World Systems — Ptolemaic & Copernican*. Translated by Stillman Drake. University of California Press.

Gandt, François. 2004. *Husserl et Galilée: Sur la Crise des Sciences Européennes*. Paris: Librarie Philosophique J. Vrin.

Garrison, James. 1986. Husserl, Galileo, and the Processes of Idealization. *Synthese* 66 (2): 329-338.

Gingras, Yves. 2001. What did Mathematics do to Physics? *History of Science* (39): 383-416.

Gorham, Geoffrey and Kenneth Waters. 2016. Introduction. In: *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century,*

23

edited by Geoffrey Gorham and Kenneth Waters. 1-28. London: University of Minnesota Press.

Hatfield, Gary. 1990. Metaphysics and the New Science. In: *Reappraisals of the Scientific Revolution*, edited by David Lindberg, Robert Westman. New York: Cambridge University Press, 93-166.

Heis, Jeremy. 2018. Neo-Kantism. *The Stanford Encyclopedia of Philosophy*, available in: <https://plato.stanford.edu/entries/neo-kantianism/#MarbScho> Consulted: 4 June 2020.

Henry, John. 2002.*The Scientific Revolution and the Origins of Modern Science*. New York: Palgrave.

Husserl, Edmund. 1970. *The Crisis of European Sciences and Transcendental Phenomenology*. Translation by David Carr. Evanston: Northwestern University Press.

Koyré, Alexandre. 1943a. Galileo and Plato. *Journal of the History of Ideas* 4 (4): 400-428.

Koyré, Alexandre. 1943b. Galileo and the Scientific Revolution of the Seventeenth Century. *The Philosophical Review* 52 (4): 333-348.

Koyré, Alexandre. 2016. *Do Mundo Fechado ao Universo Infinito*. Translated by Donaldson M. Garschagen. 4ª. ed. Forense Universitária.

Kuhn, Thomas. 1977. Mathematical versus Experimental Traditions in the Development of Physical Science. In: Thomas Kuhn, *The Essential Tension: Selected Studies in Scientific Tradition and Change*. 31-65. Chicago: The University of Chicago Press.

Kuhn, Thomas. 2012. *The Structure of Scientific Revolution*. Chicago: The University of Chicago Press.

Mach, Ernst. 1903. Space and Geometry from the Point of View of Physical Inquiry. *The Monist* 14 (1): 1-32.

Matteoli, Giorgio. 2019. Galileo, Plato and the Scientific Revolution: The Origins of Galileo's Platonism Thesis in the Historiography of Science. *Transversal: International Journal for the Historiography of Science* (7): 70-84.

Moriarty, Francis. 1994. *The Philosophy of E. A. Burtt: The Metaphysical Foundations for a World Community*. Adelaide: Thesis.

Nickles, T. 2017. Scientific Revolutions. *The Stanford Encyclopedia of Philosophy*, available in: <https://plato.stanford.edu/entries/scientific-revolutions/>. Consulted: 5 March 2020.

Palmerino, Carla. 2016. Reading the Book of Nature: The Ontological and Epistemological Underpinnings of Galileo's Mathematical Realism. In: *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*, edited by Geoffrey Gorham and Kenneth Waters. 29-50 London: University of Minnesota Press.

Parker, Rodney. 2017. The History Between Koyré and Husserl. In: Raffaele Pisano, Daria Drozdova. *Hypotheses and Perspectives in the History and Philosophy of Science: Homage to Alexandre Koyré 1892-1964*. 243-276 Cham: Springer.

Roux, Sophie. 2010. Forms of Mathematization (14th - 17th Centuries). *Early Science and Medicine* (15): 319-337.

Schuhmann, Karl. 1987. Koyré et les Phénoménologues Allemands. *History and Technology: An International Journal* 4 (1-4): 37-41.

Schuster, John. 2017. Consuming and Appropriating Practical Mathematics and the Mixed Mathematical Fields, or Being "Influenced" by Them: The Case of the Young Descartes. In: Lesley Cormack, John Schuster. *Mathematical Practitioners and the Transformation of Natural Knowledge in Early Modern Europe*. 37-65. Cham: Springer.

Shapin, Steven. 1994. Certainty and Civility: Mathematics and Boyle's Experimental Conversation. In: SHAPIN, S. *A Social History of Truth: Civility and Science in Seventeenth-Century England*. 310-354. Chicago: University of Chicago Press.

Soffer, Gail. 1990. Phenomenology and Scientific Realism: Husserl's Critique of Galileo. *The Review of Metaphysics* (44): 67-94.

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Villemaire, Diane. 2002. *E. A. Burtt, Historian and Philosopher: A Study of the Author of The Metaphysical Foundations of Modern Physical Science.* Springer.

Westfall, Richard. 1971.*The Construction of Modern Science: Mechanisms and Mechanics.* London: The Cambridge University Press.