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Article

The Mechanical Representation of Knowledge: From Descartes' Mechanized Geometry to Carnot's Heat Engine

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Abstract:

This article revisits aspects of Thermodynamics' history by applying the Cartesian conception of a machine to Sadi Carnot's work. By defining mechanically geometric curves, Descartes placed mechanical science at the centre of geometry and natural philosophy. With this, the autonomous machine - a product of this science - starts to correspond to an operational definition with the power to understand and generate new ideas in addition to a general explanation model. After nearly two centuries, Carnot presented a reversible thermal machine theory - mechanically inspired and defined operationally as an ideal model - and represented it analogically. That addressed the problem of maximizing the production of motive power. Although still limited in its ability to solve problems, it inaugurated a progressive sequence of representations with an ever-widening phenomenological reach. As suggested in this article, classical thermodynamics' emergence and development correspond to a Cartesian style theorization.

Keywords: Carnot's machine; Heuristic of representation; Descartes; Mechanical geometry

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Introduction

In general, works on the history of thermodynamics up to Clausius highlight particular aspects of this area of physics. The debate about the nature of heat and the teleological character of entropy (Callendar 1910, Newburgh 2009, Cardwell 1971, Bernal 1953) are

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examples where ontological aspects are at the core of the discussion. The role of technology and industry in the emergence and development of heat science and its relationship to human progress - based on control over nature - are other aspects traditionally addressed (Cardwell 1971, Bernal 1953).

Although the mathematical resources used and available in the 19th century to theorize in this field of Physics usually appear in historical approaches (Truesdell and Bharatha 1977, Erlichson 1999), few have explored the origins and the role of representation and its heuristics as an instrument of solving problems.[4]

The practical aspects are often contingent here. Except for the work developed by Mayer and Joule on the equivalence between heat and mechanical work (Joule 1884) - which led to the abandonment of caloric theory -, such aspects seem to be in the background in thermodynamics when we compare them to concepts and models. A possible explanation is that thermodynamics was born (with Carnot) as a field of idealizations in the Cartesian sense. The so-called Carnot reversible engine was the first and most important of these idealizations.

Regarding the relationship between industry and science, especially during the development of heat science, Bernal makes a generalization stating that "all physical science throughout the 19th century depended entirely on the support and largely on the inspiration of industry" (Bernal 1953, 5). Indeed, industry inspired the developing science of heat, raising questions and proposing ideas. Nevertheless, this differs from claiming that external factors completely shaped thermodynamics' formal and conceptual content until the 19th century. Science had a certain autonomy to develop its theories.[5] The industrial age and the new technological demands connected with it inspired and favoured the development of Thermodynamics.

There are other ways to address this issue. Here, we want to frame it according to the Cartesian conception regarding the relationship between science and technology (or industry). Yet, although this conception may be of interest mainly to Philosophy and the History of Science, this approach brings to light a theorizing dynamic that interests science. It expresses its ways of representing problems, inseparable from its solving patterns. Especially in physics, this way of theorizing establishes the heuristic range (heuristic power) of a theory and explains why the extension of this range and the incorporation of new phenomena or alternative descriptions to the original approach change the representational substrate.

First, we will understand the relationship between industry (and its technical and technology surrogates[6]) and science in the period when the ideal of mechanization emerged, that is, in the way Descartes understood it.

The Cartesian Notion of a Mechanized Geometry

The *Mathesis Universalis* project presented by Descartes in the 17th century is traditionally understood as a synonym for the mathematatization of knowledge. However, in the last few

[4] Representations and their heuristics are the core of a research program coordinated by one of the authors of this paper.

[5] Leon Rosenfeld - historian of science, physicist, and philosopher, with a Marxist bias - similarly argues that we cannot underestimate the role of artisans in the development of science. However, he criticizes Bernal for his tendency towards Marxist-Leninist dogmatism, which can only discredit the cause of historical materialism (Jacobsen 2012, 230,232).

[6] In the present context, these terms are equivalent, and, given the Cartesian framework of the problem, they are considered synonyms for *ars*.

decades, a series of works have shown the other side of this program (Arnatu 2017, Burnett 2005, Gauvin 2006, Ribe 1997), from the analysis of its evolution - initially due to the Renaissance metaphor according to which the craftsman, like Plato's Demiurge, would order the world according to a principle that would be intrinsic to him, namely, his ordered soul (*âme réglée*).

This evolution was motivated by Descartes' disappointment with the artisans of his time, including Jean Ferrier, who failed to implement his design for a machine to produce hyperbolic lenses. In Descartes' eyes, the artisans' souls were not so ordered and required rigorous training in the logic of practice. In this context, the autonomous machine replaces the Renaissance metaphor as a scientifically determined problem-solving rule where (arbitrary) human intervention is not required. Therefore, the Cartesian proposal involves the alienation of the artisans' hands and the complete automation of mechanics (Gauvin 2006, 196-199).

The basis for this review of the role of mechanical art in the Cartesian system is the redefinition of geometric curves. According to Descartes, the Greek (ancient) classification of "geometric curves" concerned only those lines whose traces resulted from the use of the ruler and compass. The curves, whose features required other instruments, received the classification of "mechanics".[7] Therefore, the mesolabe, the Nemenesis' instrument, and the Dercles compass generated mechanical curves. However, Descartes saw no sense in this classification criterion, as he considered all of these instruments to be machines:

> I am surprised, however, that they did not go further, and distinguish between different degrees of those more complex curves, nor do I see why they called the latter mechanical, rather than geometrical. If we say that they are called mechanical because some sort of instrument has to be used to describe them, then we must, to be consistent, reject circles and straight lines, since these cannot be described on paper without the use of compasses and a ruler, which may also be termed instruments. (Descartes 1996, VI, 315)

There are two moments in the above process: 1. the redefinition of geometric curves incorporating machines in their generation and thus, including mechanics as a science in the scope of geometry; 2. since it is not the intervention of an instrument that establishes the distinction between curves, it is necessary to proceed with the characterization of movement of instrument that results in one or another curve.

> It seems very clear to me that if (as is customary) we consider geometrical that which is precise and exact, and mechanical that which is not, and if we consider geometry as the science that furnishes a general knowledge of the measures of all bodies, we have no more right to exclude the more composite lines than the simpler ones, provided that one can imagine them as described by a continuous motion or by several motions that follow each other, and of which the last ones are completely regulated by those that precede. (Descartes 1996, VI, 389)

The acceptable movements that generate geometric curves must be continuous and successive, each one completely controlling its successor. The action is similar to those obtained by an autonomous machine where the gears are interconnected, and the

[7] Descartes was careful and synthetic in this classification because he recognized the Greeks' lack of clarity and unanimity (Bos 1081, 1986, 1998; Crippa 2014, Merli 2016).

movement of one determines the others' movement and the movement of the entire mechanism or system. At both moments, the geometry is mechanized, either by introducing a machine to define curves or by specifying the type of appropriate movement the machine must perform. A machine that performs this type of movement works as an autonomous machine.

It is worth mentioning that the continuous and orderly movement that governs the operation of machines generating geometric curves also characterizes the logical movement of the Cartesian method itself. According to Burnett,

> The excellent operations of reason are said to involve a continuous motion of truth-transfer by means of discrete stages in the order of reasons (...) The steps of this reasoning, like the geometry in which they are rooted, are ground in the deep metaphor of the machine, a mental model that follows mechanical guidelines. (Burnett 2005, 34)

Like the Method, our minds must incorporate this autonomous machine as a system of relationships between discrete and clearly defined parts. As a mental model, the matter that constitutes it is not physical but a pure extension, in the Cartesian style. The composition of extension and movement that explains the law of refraction in *Dioptrique*, the movement of the stars in the sky in *Meteors*, also explains the generation of geometric curves in *Geometry*.[8] However, there is a difference here: *res extensa* is not the shape of a small sphere used to model light as it passes from one medium to another but "an ordered series of relationships between discrete parts, following one another exactly and ordered to an end" (Burnett 2005, 39).

Going back to geometry, accuracy - a necessary condition for scientific knowledge - is no longer a mere synonym of simplicity expressed in the possibility of drawing curves with a ruler and compass. It is associated with continuous movements ordered constructively, modelable and executable by an autonomous machine. This reification of geometry occurs by mechanization, and the resulting geometry is mechanized. Although mechanical art is at the heart of geometry, the mechanisms that implement this mechanization are not physical; they are not physical machines; they are not linked to the possibility of building the mechanisms that implement these curves but rather to the ability to imagine such means.

It is not the physical machines that define the geometric curves but the idea of a machine. To put it in another way,

> The mechanical systems for drawing curves occupy a new place in Descartes' geometry: instead of being merely tools for generating dilapidated appearances of ideal curves, the machine is re-conceived precisely as an ideal system that generates our ideas. (Burnett 2005, 33)

Philosophically, to assert that a system generates ideas means to say that it is an epistemological instrument with heuristic value. This is the case with machines in the Cartesian conception. It is in this sense that we interpret the Carnot machine.

In the simplest case, we do not need to define a circumference as the set of points equidistant from a centre but by the movement of a compass with one fixed end and another rotating 360 degrees. Thus, we can specify the circumference and the other geometric curves by their properties and genesis, the latter related to the instrument that traces the curve.

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[8] *Dioptrics*, *Meteors*, and *Geometry* are essays published as annexes to Descartes' *Discourse on the Method*.

There are two ways of expressing the same quantity: 1. highlighting its intrinsic properties, the relationships between the curve's points, and another element (such as a point, a line, a plane, etc.). For example, an ellipse (a closed plane generated by a point moving in such a way that the sum of its distance from two fixed points is a constant); 2. Second, by genesis, the generating machine operates in continuous movement. Its neatly arranged parts regulate and create the line whose set of points corresponds to a geometric curve (an ellipse, according to the present example).

We can still speak of a third way of expressing this quantity: by the curve equation. Although without providing any proof, for Descartes, every acceptable geometric curve has an algebraic representation. Subsequently, the existence or not of an equation becomes the criterion for establishing the distinction between geometric and transcendental curves, proposed by Leibniz (Bos 1981). This third way of expressing a quantity is not independent of the previous ones. In Descartes, the algebrization of curves requires introducing the unit to reinterpret the product of line segments as another line segment, regardless of whether it is the product of two, three, or more segments. Until then, the effect of the multiplying of two line segments constituted an area; of three, a volume; more than that, it had no geometric sense.

At first glance, this might seem like a degeometrization, considering the geometry that prevailed until that moment. Descartes had something more significant in mind: to create a new geometry, an analytic geometry, without which it is impossible to establish the general connection between algebraic and geometric curves. When we think of algebraic equations representing curves, we are in a space that is no longer neither geometry nor algebra. According to this procedure, algebra is reframed by geometry and vice versa, resulting in a fusion with new heuristic resources. These features allowed Descartes to solve some classic problems that lasted for centuries; Pappu's problem is an example (Descartes 1996, VI, 377-79).

Underlying the possibility of representing the same object differently is the analysis of heuristic resources specific to each of these representations. In the endless task of problem solving, science seeks alternative representations that provide more adequate tools. Descartes highlighted this strategy by formulating geometric problems through equations, solving them algebraically, and returning to geometry.

So, in this context, there are three ways to represent a quantity: based on its genesis, property, or equation. However, it is essential to consider that representations are not just symbolic systems that express the elements of knowledge but incorporate a heuristic or operating system that aims to expand that system's problem-solving capabilities. As noted, algebraic representation (equations in the conceptual space of analytic geometry), by merging elements and features of geometry and algebra, is the way that offers more tools to solve problems.[9]

Such is the power of algebraic representation that Descartes proposes a method to solve any geometric problem in this space. First, an equation represents the problem, whose solution is its algebraic answer. Then, we return to pure geometry to order the elements (lines, segments, circles, conics, etc.) according to the solution found, which corresponds to the construction of the proof (Bos 1986). This highlights the historical relevance of geometric problems in mathematics. In ancient geometry, heuristic resources to solve such problems were scarce, leaving them almost always open. Descartes refers to this by stating that the

[9] It is essential to maintain the distinction between mechanized geometry and analytic geometry, although both are syntheses proposed by Descartes. The first refers to the resources (machines) used to delineate and define curves. The second is equivalent to the new space (Cartesian), which corresponds to a quantity's algebraic and geometric representation.

ancients did not have a general method of solving problems of this nature. In particular, faced with Pappus' problems, that is, an indeterminate problem whose infinitely many solutions form a one-dimensional locus, Descartes develops an algorithm solution that describes the problem geometrically and assumes it to be solved. Then, as analytic geometry allows the algebraic translation of geometric structures, the geometric characteristics or properties of the problem are algebraically expressed. These algebraic expressions combine known and unknown quantities, and then the unknown is isolated and determined in terms of the known. If there is more than one unknown, the number of equations available determines whether the solution consists of a point or a set of points, such as a plane curve, etc. Having found the solution, we return to geometry and arrange the elements (points, lines, arcs of circles, etc.) as determined by algebra (Descartes 1925, 21-30, Bos 1996, 313-325).

Without analytic geometry - understood as a set of translation rules between geometry and algebra - the combination of its elements is done by trial and error, and there is no method (algorithm) that leads to the solution (Descartes 1925, 17). In this sense, we argue that a representation has greater heuristic power when it has more and better tools for problem-solving. However, geometry and algebra alone, although less heuristically effective, have their relevance and interest, especially if we look at the problem from a historical perspective according to which the evolution or alteration of concepts, operations, arguments, and methods, in general, change from more concrete figures to more abstract forms.

This mathematical outcome is fraught with consequences for the Cartesian system and for physics in general. We intend to explore this second consequence here, using the thermodynamics of Carnot's thermal machine as an example. First, we formulate Carnot's problem - the construction of the heat engine that produces the most significant possible amount of driving force from the movement of caloric. As we will have the opportunity to clarify, the mechanical reduction of this heat engine enables its regular continuity, where the action of the caloric entirely controls the operating cycle. Then, the protagonist is Clapeyron, who performs the mathematization of the theory through the diagrammatic representation of Carnot's machine. Lastly - concerning the mentioned analogy - diagramming provides the inference of fundamental laws (expressed algebraically) that allow us to deduce experimental laws and rationally explain physical phenomena. However, the return to previous representations of the theory does not aim to build the solutions found algebraically, neither in diagrams nor by mechanisms. It does not seek to complete any logical movement, such as the geometric proof of the algebraic solution. Physics does not need such logical cohesion. The objective is to alter, create and abandon concepts and thus reinterpret phenomena.

Additional clarifications are necessary before moving on to the main topic. It is relevant to point out that, according to Descartes, the machines used to illustrate geometric curves' were not material but only ideas of machines. Thus, on the one hand, the problems and defects in the execution of machine designs proposed by Descartes do not directly affect his project. On the other hand, this same characteristic explains why Descartes' projects were not carried out satisfactorily in the particular case of lens making: his idea of the machine could not be translated into a material object because wood, horn, and string did not behave like the mental matter that Descartes set in motion in his mind (Burnett 2005, 64). In fact, his hyperbolic lens-polishing machine was a product of mechanized geometry, but neither Ferrier nor Huygens carried out the project.[10]

[10] In a letter dated September 8, 1837, Huygens asked Descartes to send him a small treatise on mechanics' fundamentals, illustrated with some instruments. In October of that same year, Descartes replied with the description of six simple machines (Descartes 1996, I, 435-447), whose association

And even with the implementation of a machine like this, its result would not imply, as Descartes expected, an increase in telescopes and microscopes capacity. The defects in the formation of images that the hyperbolic lenses promised to correct would arise again due to the chromatic aberrations explained by Newton in his theory of light. Then, attention turned again to the spherical lenses. This event meant the artisans' revenge due to their refusal to submit practice to theory (Burnett 2005, 120 -121).

For the above, one should not ignore the fruitfulness of Descartes' dream. It is undeniable, according to this reinterpretation, especially from *Dioptrique*, that, for example, knowledge of the law of refraction opened up the (theoretical) possibility not only of correcting defects in human vision but of increasing its potency, both for considerable distances and to tiny dimensions through hyperbolic lenses. The analysis of human vision through the law of refraction provides Descartes with the horizons for its improvement, leading him to conclude that natural and artificial organs are functionally equivalent but with different evolutionary reach (Ribe 1997, 55). Nature, exemplified in the human vision, needs improvement according to rational rules established by science; that is, nature is perfectible both in the search for the correction of defects concerning its normal functioning and in the enhancement of its power.

Modern revaluation of science is precisely that: natural human organs have evolved to provide adaptation to the environment and the survival of the species, which, in the first stages, is not yet in possession of scientific knowledge and, therefore, is subjugated to nature. Artificial organs, projected from scientific knowledge and which result in a further increase of it, put humankind in the leading position over nature.

Technology, in this scenario, must therefore be seen as an applied mathematical science (Arnãtu 2017, 108). Here there is a clear break from the previous historical period. Indeed, "for scholasticism, mechanics is a combination of geometry and physics, but geometry and physics are profoundly distinct, studying different realms" (Aunãtu 2017, 116). However, in Modernity, this distinction is abandoned and perfectly expressed in Galileo's claim that mathematics is the language of nature. In the period between the end of Scholasticism and the beginning of Modernity, the power of this assertion led to the fusion between mathematics and physics and the perception that "physics is coextensive with mathematics, the principles of geometry and pure mathematics being the principles of physics, and thus the principles of the entire Cartesian natural philosophy" (Arnãtu 2017, 117). Descartes summarizes this idea: "my entire physics is nothing but geometry (…) my entire physics is nothing but mechanics" (Descartes 1996, II, 268).

Therefore, technology as applied science is indebted to geometry and mechanics: mechanics as a machine idea generated by geometry (mechanics = geometry = physics = science) is equal to knowledge and capable of generating knowledge. In Descartes, mechanics is elevated to a status of science and not only of application as occurred in the medieval Aristotelian tradition. But, these two meanings remain in Descartes: mechanics as science and mechanics as technology. What makes mechanics a science is its identification with physics and mathematics, from which springs its ability to explain natural phenomena. Differently, when Descartes refers to the division of knowledge and puts mechanics on the same level as medicine and ethics, he means mechanics as applied science, a technology (Aunãtu 2017, 117). This argument does not intend to misrepresent Descartes's ideas in this matter (Gauvin 2006, 201). On the contrary, he is even more radical in stating "we are not sufficiently accustomed to thinking of machines, and this has been the source of nearly all error in philosophy" (Descartes 1996, V, 174).

results in more complex machines, such as an apparatus for polishing hyperbolic lenses, which he had proposed to Ferrier in 1629 (Descartes 1996, VI, 218).

Using the Demiurge image - replaced by the man himself organizing everything according to an idea of a machine - the primordial chaos is transformed into order according to it. This is not foreign to the universe that, like a machine, is composed of matter (Descartes 1996, V, 546). The Cartesian mechanism contains, at its base, extension, and movement combined mechanically, according to the idea of a machine, which orders the system while expressing the relationship between the parts and their respective movements. While affirming that "having mechanized his geometry by defining curves in mechanical terms, he came to believe that he could construct a machine out of his geometry" (Burnett 2005, 34), Descartes not only embarks on a project realizing the two-way street between geometry and mechanics - poles that remain separate and independent -, but he accounts for the original synthesis of his mechanized geometry.

Renaissance appreciation attributed to Leonardo da Vinci - according to which those who are enchanted with practice without science are like helmsmen who enter the ship without a tiller or compass, never being sure of their destination - by way of mechanized geometry and its operability in the explanation of the natural phenomena: the Cartesian project aims to explain, among other things, natural phenomena as the operation of science conceived machines understood as a synonym for geometry or physics or mechanics.

This conceptual and historical picture is relevant to understanding modernity's peculiarities, besides illustrating and illuminating some episodes of science development. In particular, is the case of thermodynamics and the role that Sadi Carnot's theory plays in it. From the industrial point of view, his reversible thermal machine was not even made because it was considered a theoretical delusion. However, as a theoretical representation capable of generating ideas, it was an instrument that aggregated and ordered the concepts related to engineering and heat science, not only of the time but afterward, with unparalleled fertility in the history of physics. It is precisely this aspect that we intend to elucidate here.

Carnot's Thermal Machine

Until the mid-1960s, Sadi Carnot was believed to be an isolated figure, a singular case in the history of science who inexplicably leaped two and a half decades ahead of his time. The historians of science Thomas Kuhn and D. S. L. Cardwell[11] were the ones who contributed most decisively to the revision of this idea, noting the debt Carnot owed to the writings of the time on energy engineering practice (Fox 2014).

From James Watt (1736-1819), for example, Carnot took the principle of the production of motive power through the expansion of steam and the need for a separate condenser which, in the ideal engine, transforms into the cold body or heat reservoir. This last technical detail enabled him to implement the condition of maximum production of motive power by analogy with the hydraulic machine. The context of this analogy exports the concept of reversibility to the science of heat. Carnot had learned this very idea of an ideal hydraulic mechanism that transfers all the power from falling water into a driving force from the works of his father, Lazare Carnot (1753-1823)[12]. Carnot's merit and genius brought all these

[11] Cardwell's central aim in "From Watt to Clausius" is to criticize the idea that "science was established in the 17th century and that everything that followed this period was stacked up almost automatically, as an inevitable progress, thanks to the application of the scientific method in the framework established by Newton" (Cardwell 1971, p xi). He does not take into account, however, that another personality of the 17th century also formulated an even more comprehensive theory of science, in which thermodynamics does not appear as a foreign body in a sense we will show here.

[12] According to Gillespie and Pisano, there was a respectful and substantive relationship between the work of father and son. Sadi applied the analysis that his father had developed in his study of ordinary

elements together in a coherent argument (Cardwell 1971, 192-200).

These principles and practices were not only part of the general heritage in the field of power engineering of the era, but the central question Carnot sought to answer in his *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance* and also belonged to this scope of technological knowledge (Carnot 2005, 6). However, the method and the intention he applied, enabled him to develop a theory that led to the establishment of a new science, thermodynamics. But the rationalization of heat engines required the adoption of a conception of the nature of heat. In Carnot's time, heat was predominantly seen as a substance, called the caloric, which could not be created or destroyed, nor transformed into mechanical work or in what we recognize today as other forms of energy. To this extent, heat engines did not turn heat into work or vice-versa; they didn't transform anything! This point is crucial. For Carnot, heat engines functioned as a carrier of the caloric, communicating heat from a source to a reservoir, maintained at different temperatures. It is against this background that we should read the statement that the production of motive power is then due not to the actual consumption of caloric, but to its transportation from a warm body to a cold body. This expresses Carnot's commitment to the substantial theory of heat in *Réflexions*.[13]

The machine must receive heat from the hot source and transmit it to the cold source (reservoir) using an intermediate substance. Then along with the substance, the machine is moved away from the hot source and brought into contact with the reservoir. At this point, the substance transfers heat to the reservoir. Continuing the process, the cycle is repeated, responding to a requirement of continuous operation. The generalization of the circumstance of power production lets Carnot state:

> Wherever there exists a difference of temperature, motive power can be produced. Reciprocally, wherever we can consume this power, it is possible to produce a difference of temperature, it is possible to occasion destruction of equilibrium in the caloric (Carnot 2005, 9).

It is in this context that Carnot recognizes that a necessary condition for the maximum production of mechanical work is that "in the bodies employed to realize the motive power of heat there should not occur any change of temperature which may not be due to a change of volume" (Carnot 2005, 13). This condition, which we will call assumption 1 or principle of expansion, is inspired by Watt's expansion law and immediately becomes a principle: [this condition] "should never be lost sight of in the construction of heat engines; it is its fundamental basis. If it cannot be strictly observed, it should at least be departed from as little as possible" (Carnot 2005, 13). In fact, if the engine and the source are at the same temperature during contact, the transferred caloric would lead to the expansion of the working fluid without increasing its temperature. Another thing is that,

> Since every re-establishment of equilibrium in the caloric may be the cause of the production of motive power, every re-establishment of equilibrium which shall be accomplished without production of this power should be considered as an actual loss. Now, very little reflection would show that all change of temperature which is not due

machines' operation to the functioning of heat engines. Specifically, Sadi's idea of a reversible process originated in his father's use of geometric motions in the analysis of machines in general (Gillespie and Pisano 2014).

[13] For convenience, in this article, power, motive power, driving force and mechanical effect are all similar expressions of what is today understood as mechanical work.

> to a change of volume of the bodies can be only a useless re-establishment of equilibrium in the caloric. (Carnot 2005, 12-13)

The connection between temperature variation and volume variation - with the consequent production of motive force - enables the conception of a heat machine from a mechanical machine. In other words, assumption 1 implements the mechanization of the heat engine, in the Cartesian sense, which is different from reducing the science of heat to Newtonian mechanics since the temperature variation is said to be equivalent to the variation in volume: it generates one variation from the other and vice versa: the temperature variation is not reduced to the volume variation. If this were the case, the former could be excluded and described as the most fundamental variable. But it is not. The phenomenology of the heat engine cannot do without the concept of temperature and its variation. According to our understanding, this mechanization is Cartesian.

By demanding such a close link between variations in temperature and volume, Carnot parameterizes the thermodynamic quantity par excellence by a quantity translatable as an extension. Thus, the temperature variation starts to be manipulated by continuous movements of clearly defined discrete parts, understood not as physical but as geometric parts. Therefore, mental modelling under mechanical guidelines is analogous to that of Descartes' lens-polishing machine. Similar to how the fusion of geometry and algebra gave rise to analytic geometry, which has properties and resources that the parts in isolation did not have, the fusion of thermology and mechanics (geometricized mechanics) engenders a new area of knowledge with its laws: thermodynamics. Therefore assumption 1 is the (scientific) principle that should guide the construction of the thermal machine and establishes a heat science geometrized by the equivalence between temperature and volume variation. But, the above-stated principle is an idealization, and it is impossible to avoid such waste in a real machine.

When these processes are represented mechanically, assumption 1 requires that the piston container be removed from the source after the isothermal expansion and continue to expand adiabatically until the temperature drops to a value equal to the reservoir; only then are the two brought into contact. Therefore, the condition that the caloric transfer entails just a variation in the working fluid's volume also imposes limits on the adiabatic expansion or contraction of the fluid. However, this second phase of expansion occurs because of inertia: the isothermal transfer of heat during contact of the source with the engine produces an expansive movement in the volume due to the substantial nature of the caloric, that tends to continue even after the removal of the container. When the temperature drops to the reservoir value, the container is in contact with the latter. A volume contraction will occur from then on with the transfer of caloric from the working fluid to the reservoir.

But what amount of caloric is allowed to flow in this direction? Here the caloric theory is of use once again. The quantity is precisely the one transferred to the machine by the source. This way, the amount of heat in the working fluid at the end of the isothermal contraction movement is identical to the initial state before contact with the heat source. The container is then removed from the reservoir. Still, its volume will continue contracting adiabatically, causing an increase in temperature until a final value, which, given the system's initial condition, is the temperature of the heat source. Carnot's engine operates this way (Carnot 2005, 10,18).

Engineering is not the source for the operations that intercalate the isothermal processes and allow for the cycle's closing. In 1816, the engineer Robert Stirling patented a closed cycle machine where two isometric processes intersperse the isothermal processes. Carnot was unaware of this work (Mendoza 1976, 180). Like the Carnot engine, the Stirling engine also satisfies assumption 1, showing that this principle can be cyclically schematized in other ways. It is a principle subject to a diversity of mechanical schematizations.[14] The important thing is that the increase in volume implies a precise and unique drop in the temperature of the working fluid, corresponding to the temperature of the heat reservoir. This link makes the system reversible: variation in volume generates a temperature change; conversely, the same change in temperature produces an identical shift in volume.

Back to the intercalation of isotherms by adiabatics, the source refers more to the theory of gases. According to Robert Fox, Carnot introduced the adiabatic processes that interspersed the isothermal dilation and contraction under the influence of Clément and Desormes, whom he knew personally. They calculated the motive power produced by an adiabatic process, assuming that steam would obey the law of Mariotte and Gay-Lussac (Fox 2014, 242-245).

Clément, Desormes, Carnot, and others inaugurated and established a way of studying steam engines that was detached from strictly technological activity. For example, in his analysis of the principle of expansion, Hachette, whose works were widely known in France in 1820, considers the immediate problems of engineering practice particularly. That is, those relating to the functioning of actual heat engines and not those in ideal conditions. Clément, Desormes, and Carnot, seem to consolidate a tradition

> in which the abstract style of Carnot's treatment of an idealized engine was by no means out of place: When Clément, Navier and Charles Dupin noted the huge disparity between their predictions and the actual performance of the engines they were discussing, they did so with no sense of failure, for like Carnot, but unlike Hachette, for example, they were seeking to define a theoretical maximum which they knew was unattainable (Fox 2014, 166-167).

Their intention was scientific, but Carnot was the first to propose its complete scientific rationalization. His rationalization of steam engines serves as the Cartesian stratagem for reducing the unknown to the known. It is the understanding of the heat engine as a mechanical machine that produces motive power as a water wheel would from dropping water.[15] In the hydraulic engine, the motive power produced depends on the amount of water and the height of the fall. Analogically, in the heat engine, the production of motive power depends on the quantity and on the height of the drop or transport of the caloric, which is the difference in temperature between the source and the heat reservoir.[16]

By reversing the operation of the water wheel, meanwhile, it is possible to raise water to its original height. So by performing work on the heat engine, it is possible to return the caloric to the heat source. In other words, according to Carnot, the ideal heat engine must be reversible. As such, two of these engines can be coupled, one working in one direction and the other in the reverse one; whatever the first produces in motive power, the second one uses to return caloric to the heat source, resulting in a continuous movement, but without production of mechanical work external to the coupled system.

Although assumption 1 already indicates the possibility of reversibility, the analogy

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[14] Although conceived before the Carnot engine, the Stirling engine was not presented as a machine that maximizes the production of mechanical work; this conceptual achievement is Carnot's merit.

[15] Lazare Carnot, the father of Sadi Carnot, presents a reversible hydraulic machine in (Lazare 1803). As a military engineer and mathematician, he continues Cartesian project and offers mathematical formulations of general principles that govern the operation of mechanical machines. Even in this sense, the link between Sadi Carnot and the Cartesian project is very close machines.

[16] According to this analogy, the caloric plays the role of the water substance whose quantity is conserved. Differently, in the energy paradigm, the heat must be associated with an accidental quality of water that is partially transferred to the wheel.

with the mechanical machine allows the concept of reversibility to be imported into the heat engine. Besides, the reversibility is equivalent to the admission of perpetual-motion-type systems. This is not a problem for Carnot, as it was also not for Mechanics even before Newton's *Principia*. This way, the answer to the question about how much caloric can be recovered from the reservoir by applying a certain amount of motive power becomes simple: the same amount that, flowing, would produce this motive power. In heat engines, reversibility follows as a result of the principle of least waste; that is, in the substances used for creating motive power from heat, there is no change in temperature other than the one due to a volume change. In this ideal regime, it is possible to manipulate the values of one of these variables (temperature or volume) using exclusively the other as a control variable. In a nutshell, reversibility meets the requirement for the operating scheme of the ideal thermal engine, inspired by a mechanical machine, which is irreversible but taken purposely as reversible.

The question about the role of the working fluid in the ideal engine arises. With the theoretical substrate at hand, this question is settled quickly. The coupling of two reversible machines - the first one functioning to produce motive power from caloric transport and the second one using the motive power of the former and returning caloric to the heat source - allows for the (ideal) implementation of a perpetual motor where neither an external motive force to the coupling is produced, nor caloric is accumulated at the source or in the reservoir. But if the efficiency of the reversible heat engine depends on the nature of the working fluid employed, one case or another may occur because the use of a better-working fluid could further improve the efficiency of the reversible heat engine. In his *Réflexions*, Carnot emphasizes only the case of the unlimited production of motive power, presenting the following conclusion:

> This would be not only perpetual motion but an unlimited creation of motive power without consumption either of caloric or any other agent whatever. Such a conception is entirely contrary to ideas now accepted, to the laws of mechanics and sound physics. It is inadmissible. We should then conclude that the maximum of motive power resulting from the employment of steam is also the maximum of motive power realizable by any means whatever. (Carnot 2005, 11-12)

This conclusion is commonly expressed by stating that the Carnot machine has the highest efficiency among all thermal machines. This argument is not based on efficiency as the quotient between the energy obtained and the energy supplied, but on the prohibition of unlimited creation of motive power. In principle, his commitment to the caloric theory would not allow him to follow this path since the quantities involved in this quotient are of a different nature. However, Clapeyron, either by mistake or audacity, proposed this quotient and gave rise to the process of the mathematization of Carnot's theory.

The level of detail in the previous paragraphs sought, among other things, to emphasize the scientific assumption used by Carnot in the construction of his theory and characterize it according to the mechanization of knowledge proposed by Descartes. In addition to the expansion principle, which is presented below according to Carnot's formulation, there are still three other principles: (1) that any temperature variation occurs as a result of the variation in volume; (2) the analogy with the hydraulic machine and (3) the prohibition of unlimited creation of motive power without consumption of either caloric or any other agent. These assumptions articulate the theory of a heat engine's maximum production of motive power and establish the reversible machine as a thermodynamics principle.

A heat engine of mechanical inspiration is now at the heart of the science of heat: it is, indirectly, the mechanization of that science, as we will justify better in the next section.

From this viewpoint, the Carnot machine generates knowledge by bringing a phenomenology of the generation of driving force through the transport of caloric. It is an extension of the caloric theory. In parallel, this level of detail also allows us to analyze the logical role of the caloric hypothesis in Carnot's theory.

According to Klaster, the hesitant use of caloric theory by Carnot only comes to light when reading his letters and manuscripts (Klaster 1976). Although rarely mentioned in *Reflexions*, the adoption of the caloric theory is explicit. It is part of the argument that leads to the enunciation of the ideal heat engine principle. This principle states there is no heat engine operating between the exact temperatures of the source and the reservoir that can have a better performance than the Carnot engine. Furthermore, every heat engine with a reversible operating cycle is an ideal engine that maximizes power output.

Given the subsequent development of the science of heat, the above principle would be the second law of thermodynamics in Carnot's version. His first law, in turn, would not be that of conservation of energy, since he had not conceived the equivalence between heat and work, but that of the conservation of caloric. The combination of both principles results in the theorem: "The motive power of heat is independent of the agents employed to realize it; its quantity is fixed solely by the temperatures of the bodies between which is effected, finally, the transfer of the caloric" (Carnot 2005, 20). The agent responsible for producing motive power is the working fluid. And since motive power is dependent only on the movement of caloric,

> (...) When a gas passes without change of temperature from one definite volume and pressure to another volume and another pressure equally definite, the quantity of caloric absorbed or relinquished is always the same, whatever may be the nature of the gas chosen as the subject of the experiment. (Carnot 2005, 22)

Mathematically, the caloric theory allows us to identify the quantity of heat of a body as a property, that is, to describe it as a state function (Lervig 1976).

Following Carnot's reasoning, there is no doubt that caloric theory plays a role in his work of aligning engineering with the theory of gases and Watt's principle in the form of an ideal heat engine. This fact reinforces the impression that *Réflexions* can be an analysis regarding the nature of heat and its application in the conceptualization of a heat engine. Finally, the conclusion is that the nature of heat is such that its transport from a hot body to a cold one can result in the production of motive power, which is maximized under the condition of reversibility of the operating sequence of the heat engine. In this scheme, the independence of the engine's efficiency concerning the nature of the working fluid finally becomes necessary.

Analyzing the relationship between work and heat in the context of steam engines required incorporating a theory about the heat. Although the theory chosen by Carnot in *Réflexions* was abandoned two decades later and replaced by the mechanical interpretation, it works as an auxiliary hypothesis in the set of principles (1), (2), and (3). As an auxiliary hypothesis, the caloric can be replaced by another hypothesis without abandoning Carnot's theory. Initially, not everyone understood the logical link between Carnot's heat engine theory and the caloric.

Heat theory imposes an interpretation on the nature of the operation of the heat engine. If one adopts the caloric theory, the engine is interpreted as a heat conveyor pump, which carries heat from a source to a reservoir. This is, moreover, Carnot's interpretation. If one subscribes to energetic theory, the machine works by converting one type of energy, heat, into another, mechanical work. The auxiliary hypothesis regarding the heat theory sheds an interpretative light on the general argument of the ideal heat engine. The engine must be a conveyor or converter because, as an engine, its operation needs to be specified, and to this end, a heat theory - with its metaphysics and phenomenology - must be engaged. In other words, the theory of heat provides semantics to the heat engine. Without that, the thermal machine is pure syntax. In this sense, Truesdell's attempt to operationally build classical thermodynamics from simple and natural assumptions about heat engines - developed by simple and rigorous mathematics, with no physical arguments and no appeal to metaphysics, following the axiomatic thermodynamics of Carathéodory, has been fallacious from the beginning, since he considers the thermal machine to be a convert. Metaphysics, which he says is unnecessary, quietly underlies the thermal machine's operability (Truesdell and Bharatha 1977, 578).

The Analogic Representation of the Carnot Machine

Of the three Cartesian modes of representing a quantity, in the genesis mode, the machine operationally defines a curve and then realizes it as an idea in the realm of mechanized geometry. The concept of a geometric curve, following the idea of an autonomous machine, must be incorporated into our minds as a system of relationships between discrete and clearly defined parts (Burnett 2005, 33, 39). This system of relationships must be such that each movement determines its subsequent one. It is not enough for the machine components to be connected. It is necessary that, in this mechanism, the pieces move relatively according to a specific order of successive determinations.

The Carnot engine, due to the constraint that no temperature variation does not result from volume variation - illustrated in the previous section -, makes the thermal engine that works according to this constraint (principle 1) correspond to a system of relationships between discrete and clearly defined parts: the movement (the change in the volume of the container) initiated with the caloric transfer is continued (following principle 1) until the fluid temperature reaches the value of that of the reservoir. The expansion is interrupted due to contact with the reservoir, and then the contraction is started. Thus, the first movement produced by the caloric transfer is orderly regulated, the later movements being defined by the previous ones according to the rule imposed by principle 1: movement of the caloric moves the volume of the container; the volume variation generates a drop in temperature and the contact with the reservoir with the restart of the caloric movement but now with a decrease in the volume of the container, and so on until the system returns to its initial conditions.

The fact that the role of water in a water wheel inspires the action of caloric in the Carnot engine contributes to the mechanical understanding of the element that controls the ordered sequence of movements of the parts that make up the machine. Although invisible and subtle, the action of caloric to initiate and determine the movements of the piston of the container that contains the working fluid is real and sensitive.

It is as if the first movement of caloric univocally generates, following the constraint of principle 1, the following movements until the system returns to its initial state. This is the meaning of the analogy: machines that generate acceptable curves correspond to clearly defined discrete parts in which their movements are regulated; in the Carnot engine, components (furnace, container with working fluid and piston, heat reservoir) of heat engines move by the transfer of heat governed by principle 1.

Following his argument - when Carnot establishes the analogy with the hydraulic machine (principle 2), importing the notion of reversibility and implementing a reversible heat engine and, finally, stating that mechanical work does not arise from anything (principle 3) - the property that makes his machine the most efficient is reversibility. By emphasizing this property, a study can be carried out in which the heat engine components become unnecessary. Components or discrete parts and their movements, mechanisms, and structures disappear when the machine is reduced to its reversible cycle, referring to the sequence of pressure and volume values the gas is subjected to.

As the set of points equidistant from a central point or the drawing made with a compass provide us with the characterization and geometric properties of a circle, the reversibility or diagrammatic representation of the values of pressure and volume of the substance along an operating cycle can provide us with the characterization and properties of the Carnot engine.

Carnot's machine corresponds to a scientific project built on mechanical and metaphysical assumptions, the roots of which go back to the Cartesian notion of a machine. Such assumptions are the principle of expansion (1), the ideal hydraulic machine model (2), and the restrictive condition of limited driving force creation (3). These principles made it possible to implement a sequence of processes in a thermal machine with maximum efficiency. Therefore, the Carnot machine is an operational definition of maximizing driving power. Thus, in the same way that mechanical systems for drawing curves have taken a new place in Descartes' geometry, the Carnot machine has acquired the status of an ideal system that generates new ideas about the production of driving force by heat, realizing the idea of maximum performance and originating thermodynamics as a science.

This ideal system is a theoretical model represented in terms of the elements inherent to thermal machines. While in Descartes' mechanized geometry, this way of expressing a quantity was called representation by genesis, we propose that its equivalent for thermodynamics be called analogic representation.[17] The problem of maximum production of motive power is formulated, articulated according to the heuristic resources provided by engineering, physics, and mechanics specific to that representation, and solved there. The development of the problem requires auxiliary hypotheses, including the caloric theory.

Understanding representation as a conceptual space with problem-solving capabilities is equivalent to understanding it as a language to write an algorithm capable of accomplishing a specific task. At first, the algorithm - with problem-solving capabilities that originated from the confluence of several fields of knowledge (heat engine engineering, heat metaphysics, gas science, etc.) - solves the motivating problem of *Réflexions*, corresponding to Carnot's machine processes. But Carnot recognizes that his machine is only one of several whose operation would result in the maximum driving force production. Suppose we identify a machine that performs the maximum production of mechanical work. In that case, the general property corresponding to the original problem's solution will emerge by inference (not induction). And with this principle, other thermal machines with the same purpose can be designed. In this representation, however, there are no resources to express the machine's efficiency mathematically and experimental laws concerning the change of state of substances.

The mechanical and metaphysical assumptions mentioned above make it possible to carry out Carnot's project of designing a machine that maximizes motive power production. They also identify this machine as reversible, characterizing an ascendent logical movement. While the Carnot machine is specific, reversibility is a general property. Even though initially understood in another context, the Stirling engine is also reversible. Maximum production of driving force and reversibility become synonymous, identifying the latter as the property of efficient thermal machines. Here too, only science can propose and design efficient

[17] We have already commented that representation by the property, in thermodynamics, is made by the diagrammatic representation. We will not have the opportunity to explore the heuristic resources that this representation has and its role in constructing the science of heat. In a forthcoming paper, when we present the analysis performed by Clapeyron, we will see that the Carnot engine will be devoid of its mechanical elements, leaving only the cyclic sequence of thermodynamic processes represented diagrammatically. Here the operation of the mathematization of this science begins.

machines.

On the other hand, we can create other efficient machines according to fundamental principles, but now in a downward logical movement based on the general property (reversibility). Like Descartes, for whom the representation by the property is suitable for algebraization, reversibility - the property that maximizes the efficiency of a heat engine - is also. Clapeyron will carry out this work.

Despite other programs that implemented reversibility, none was as fertile as the one proposed by Carnot, which stimulated and promoted the birth of classical thermodynamics. Although represented in different conceptual spaces, these programs run a more or less reliable version of Carnot's program. Different representations - with additional heuristic potentials - responded to new problems and expanded the thermodynamics' phenomenological reach, including the changes in the state of matter and the "critical points" 'of Thomas Andrews. In the analogic representation, the Carnot machine is the one that performs the maximum production of driving force with reversibility as a condition for this realization. Subsequently, in the face of new problems, this representation proved limited in its heuristic resources and was replaced by another. Clapeyron's work, with a diagrammatic representation of Carnot's theory, is an example of this.

Given the fundamental role of the Carnot machine - understood as an ideal model - the construction of thermodynamics occurred in a similar way to what we saw in other areas of physics: Newtonian mechanics (with the use of point mass models and the principle of inertia), the kinetic theory of gases (with the ideal gas model), quantum physics (in its initial stage - with the modelling of radiation absorption processes in spring-mass systems), etc. Thus, idealization is an intrinsic part of the construction of science, functioning as a model of reality and reconstructing it through logical demands. In the case of thermodynamics, the model of a machine is at the origin of this reconstruction.

As suggested in this article, classical thermodynamics - more than any other branch of physics - corresponds to a theorization in the Cartesian style, understood as the mechanization of nature, that is, an understanding of nature as a machine (scientifically) formulated from mechanized geometry. According to a tradition that goes back to the ancient Greeks, geometry shapes rationality, first in its most logical, deductive aspect, as in Euclid's Elements. Then, as in Descartes, in its most methodological aspect - the method as the ordered movement of a machine. Thus, knowledge and nature are understood in mechanical terms. The need for ordering is such that "if we cannot discern an apparent order, we have mentally to forge one by the power of *cogitatio*" (Gauvin 2006, 192).

Before, geometry was the language of nature. With mechanized geometry, machines started to work as methodological guidelines to discover the laws of nature. Machines have become an epistemological instrument. The Carnot engine is the heat engine with the highest possible efficiency, which forges an order, a kind of law of nature. However, in the future representations that this theory assumed, this principle acquired more and more abstract forms until arriving at the second law of thermodynamics as enunciated by Clausius: the universe's entropy tends to the maximum (Jaynes 1996).

Like Descartes' projects, the Carnot machine is the result of scientific knowledge and generates scientific knowledge. However, given the need for the mathematization of this theory, Carnot's analogic representation becomes limited. Another representation is necessary. Other scientists will be in charge of unfolding this instrument's heuristic fertility. However, such fertility will come from changing the representation of the problem and not altering the reversible machine's mechanism or the underlying heat theory. The unfolding of this heuristic potentiality will be the subject of a forthcoming paper.

Conclusions

The contextualized reconstruction proposed here - based on the Cartesian *Mathesis Universalis* of a mechanized geometry - and applied to Carnot's work allowed us to identify scientific intentions materialized in the idea of the perfect heat engine and the theory of heat. From a logical point of view, this commitment emerged as an auxiliary hypothesis preceding propositions of a general character, which explains why the theory of the ideal heat engine and its consequences survived even with the decline of the materialist theory of heat.

We consider the role of representations central in this work - with their intrinsic heuristic resources - and the internal dynamics established between them in developing a theory aiming to solve problems or incorporate new phenomena.

In this fashion, Carnot's machine is a theoretical representation that incorporates the engineering of its time, the concept of caloric, and the empirical laws of the behaviour of gases, articulating them and establishing the reversibility as the condition of the maximum production of motive power.

Mechanical assumptions, in the form of idealizations, derive the condition of reversibility, transforming the art or technique of building heat engines and their related areas into the science of thermodynamics. In this sense, reversibility corresponds to the condition that summarizes Carnot's solution to maximize the motive power obtained from heat. Here, we emphasize Carnot's originality in recognizing that it is the theory of a heat engine and not a substantialist view of heat and its properties that solves the proposed problem; that is, the space of representation of heat engines - with the use of specific terms for this space, such as source or reservoir of heat, gas or steam, cylinder, piston, etc. - articulates the solution of the problem.

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