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Article

Michael Scot and the Music of the Spheres

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Abstract:

Herein, it is shown that Kepler's contribution involving the "Music of the Spheres" can be traced back to Pythagoras and Ibn Arabī through Michael Scot. We find threads linking this body of work associating music with astronomy leading to the modern and extensive astronomical subject of orbital resonances. In particular, we find that Fibonacci numbers play a significant role in this context.

Keywords: Middle-ages; Ibn Arabī; music; astronomy; Fibonacci numbers; orbital resonances

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Introduction

Before Johannes Kepler gave us his famous astronomical "laws", he attempted to explain the relations between astronomical bodies in terms of Pythagorean geometry. His *Mysterium Cosmographicum* (Kepler et al. 1596) proposed a model for the solar system in which the distance proportions between the six planets known at that time, could be understood in terms of the five Platonic solids (Ball and Coxeter 1987). This model gave reasonable numbers at the time but has long since been discarded.

However, Kepler made another attempt, in the context of the natural world with music. The central set of "harmonies" known as the *Musica Universalis* or "music of the spheres" (Caspar 1993) inspired his 1619 book *Harmonices Mundi* (Kepler 1998). This model regarded proportions in the movements of celestial bodies as a form of music. In a paragraph towards the end of his 1611 essay "On the Six Cornered Snowflake" (Schneer 1960), Kepler speaks of this music in terms of ratios of tone intervals about the golden number which has a well-known connection to the Fibonacci sequence (Scott and Marketos 2014b). Kepler

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began by exploring regular polygons and regular solids, including what became known as Kepler's solids (Ball and Coxeter 1987), which were related to his earlier geometrical work.

One may ask, since Kepler's early geometrical model was abandoned, why bother with this examination? The answer is that although Kepler's notions of using proportions made of rationals (fractions of integers) to model proportions of distances in orbits does *not* work for the planets of our solar system, they do work for e.g., Jupiter's moons and Saturn's moons. They also work for planets *outside* our solar system, i.e., exoplanets (Yaqoob 2011), (Tony C. Scott, Therani, and Wang 2017, 3.1.6). They are called *"orbital resonances"* and are a central theme in celestial mechanics (Lemaître 2010). We also find that Fibonacci numbers appear in these orbital resonances (Pletser 2019), thus demonstrating that what many would insist is faulty reasoning can nonetheless lead to impressive results. It is instructive to trace back the influences that lead to Kepler's model.

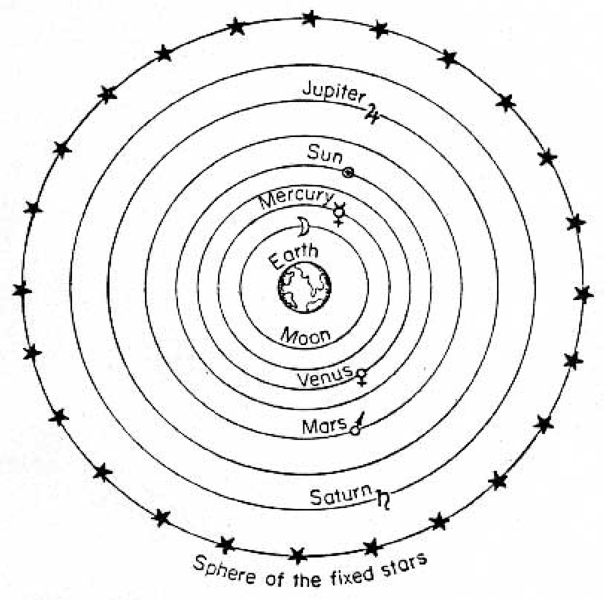
Many creators of Physics curricula are still embarrassed to admit that Kepler was an astrologer forgetting that in ancient times and the middle-ages, the discipline of astronomy and the notions of astrology were joined together. The Nobel-Prize winning Physicist, Wolfgang Pauli, taught himself Latin to such a meticulous degree so as to fully understand Kepler's writings (Jung and Pauli 1955). For example, Pauli found that the number 3 (a Fibonacci number), was more fundamental for Kepler than the number 4, as emphasized by a competing astrologer. As given in chapter 5 of his *Harmonices Mundi*, Kepler's attempt at finding harmonies within the solar system eventually lead to his famous (and successful) third law which uses successive Fibonacci numbers 2 and 3 in the ratio R^3/T^2 where R is orbital radius and T is the orbital time (Caspar 1993).

Kepler might have learned about the Fibonacci numbers from his mother, Katharina, who sold herbal medicine. She was accused of witchcraft and Kepler himself had a devil of a time (no pun intended), saving her from being burned at the stake (Caspar 1993). Herein, we show that the notion of a link between music and astronomy can be traced back to Pythagoras. We will also show how it was extended by the Sufi philosopher Ibn Arabī, and passed on to his contemporary, Michael Scot (T. C. Scott and Marketos 2014a).

From Pythagoras to Ibn Arabī

Dunja Rašić gives much information about Pythagoras and Ibn Arabī. As she points out (Rašić 2022):

> The story goes that Pythagoras of Samos (d. 495 BC) walked past a blacksmith's shop in the sixth century BC. The sound of hammering soon caught his attention; with the blacksmith explaining to him how the weight of a hammer impacts the sound it makes. This encounter presumably inspired Pythagoras to develop his theory of *Musica Universalis*, which later came to be known as the music of the spheres (*Musica Spherarum*) …



Music of the Spheres - Courtesy George N Gibson, UConn

In the Pythagorean concept of the music of the spheres, the interval between the earth and the sphere of the fixed stars was considered to be a *diapason* (Pythagorean interval) (Webster 2023, diapason) – the most perfect harmonic interval. The frequency ratio of every Pythagorean interval is a ratio between a power of two and a power of three confirming the Pythagorean requirements that all intervals be associated with ratios of whole numbers.

The following arrangement is most generally accepted for the musical intervals of the planets between the earth and the sphere of the fixed stars: In terms of distance from the center (Earth), the progression is as follows:

Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, Fixed stars (1)

i.e. 7 "heavenly bodies" beyond Earth, as shown in Figure 1. Beyond Saturn, more than one sphere can be considered. Ibn Arabī believed God created the 7 spheres, each with a planet "swimming" in each of them alongside angels. Rašić explains how the Pythagorean concept of the Music of the Spheres was integrated by Akbarian Sufism for Sufi gatherings. Therein, the question arises: "how can people hear this music from the planets?":

> Through some unutterable, almost inconceivable likeness to gods, [Pythagoras's] hearing and his mind were intent upon the celestial harmonies of the cosmos. It seemed as if he alone could hear and understand the universal harmony and the music of the spheres of the stars which move within them, uttering a song more complete and satisfying than any human melody, composed of subtly varied sounds of motion and speeds and sizes and positions. (Rašić 2022, sec. 3)

Kepler himself did not believe this "music" to be audible, but felt that it could nevertheless be heard by the soul. Ibn Arabī considers the ability to hear the music of the spheres a question of level of spirituality and formulated practices and customs to help Muslims reach that level. Ibn Arabī's ideas concerning the Music of heavenly bodies would reach Kepler in Christian Europe through an intermediary, namely Michael Scot.

From Ibn Arabī to Michael Scot

Ibn Arabī was born in Spain and spent time in Andalusia and Morocco until the 1200s to arrive in Mecca in 1202 CE and finally settle in Damascus by 1224 CE. Conversely, Michael Scot was in Toledo, Spain in the early 1200s and perhaps earlier (Burnett 1994) and he might have reached Morocco (Tony C. Scott 2017). Both were interested in the scholarly work of Ibn Rushed (Averroes) (Arnaldez 2000). Thus, they were contemporaries in time and space and could have actually met each other. Recognized primarily for translating scholarly works from Arabīc to Latin (Thorndike 1965; Haskins 1927; Burnett 1994), the contributions of Michael Scot have been underestimated. We cite a few that have only come to light, relatively recently:

1. Charles Haskins had dismissed a very detailed description in Latin concerning the medical case known as "Mary of Bologna" as a "calcified fibroid tumor" (Haskins 1927, 274). However, in the early 1970s, the exacting detail of Scot's description enabled a new and different medical diagnosis: this was a very rare case of miscarriage or "spontaneous abortion", not followed by immediate expulsion, of twin embryos, dead at different dates and calcified (O'Neill 1973, 1974). In short, Scot had actually recorded, back in the 13^{th} century, a rare medical case and this has not been fully appreciated until the 20^{th} century!

2. An analysis on the origins of the Fibonacci numbers (T. C. Scott and Marketos 2014b) suggests that Michael Scot may have played a role in the formulation of the Fibonacci sequence which would explain why Leonardo de Pisa dedicated the second version of his famous book"Liber Abaci" to him.

3. Michael Scot is quoted in a meteorological description of four parallel rainbows in the sky (Thorndike 1965, 69). Thorndike stated "the quaternary rainbow arc is not known to appear in nature". However, a recent analysis (Tony C. Scott 2017) has shown that quaternary rainbows and various types of multiple rainbows have been observed. Furthermore, this work suggests that Michael Scot might have obtained this folklore from the Tuareg, indicating that he ventured south of Toledo and reached Morocco.

Herein, we extract yet another contribution from Michael Scot. An analysis by Christian Meyer of the music and astronomy in Michael Scot's *Liber Quatuor Distinctionum* (Meyer 2009) mentions the Persian astronomer Alfraganus (Al-Farghani) (Abdukhalimov 1999) who asserted the Universe was made of 1022 stars. Michael added to that the 6 known planets and our sun, thus the 7 heavenly bodies of (1) in Figure 1 acting as the musical notes of an octave, which in "Solfège" mode (Webster 2023, solfege) is expressed with the familiar *Do, Re, … Sol, la Te*, as initiated by *Guido of Arezzo* in the 11^{th} century (Webster 2023, Guido of Arezzo). This adds up to $1022 + 7 = 1029$ bodies which can be readily factored as $1029 = 3 * 7^3$. This factorization around the prime numbers 3 and 7 would at least have the merit of explaining Michael Scot's reference in his *Liber Quatuor Distinctionum* to the three octaves/registers of bass, treble and super-treble (altissimo), each of which ideally includes seven tones/notes. Unlike 3, the number 7 is not a Fibonacci number, but it is nonetheless a Lucas number prominent in many belief systems including all three of the Abrahamic religions.

Meyer states little was done in the (European) Middle Ages to associate musical tones with astronomical patterns. Thus, what Michael Scot did was unusual. As Meyer states:

> The presence, in this book of cosmology and astronomy, of such considerable developments on music is relatively surprising, because this discipline is virtually absent from medieval astronomy treatises. If music indeed maintains close relations with astronomy in Late Antiquity and, in particular, in the second book of Harmonics of Ptolemy, the links between these disciplines were distended from the High Middle Ages … Besides, the diffusion of Aristotelian thought which supports, on this point, the idea that the planets move in silence, undoubtedly contributed to the weakening of this theme. (Meyer 2009, 121)

Here, planets follow the same order as in (1) for Ibn Arabī with an additional 8^{th}, 9^{th} and 10^{th} spheres. The 8^{th} is denoted *firmamentum stellarum* (the support of the stars) while the 9^{th}, *caelum empyreum* (highest heaven). He also adds 9 semi-tones within the 10 tone scale for a grand total of 19 semi-tones. Note that this scale was not used by musicians of the middle-ages. Scot was idiosyncratic in his approach to music and that included his own invented terminology for musical instruments, based apparently, on Jewish gematria (Tony C. Scott 2017).

The analogy between sounds and planets leads Michael Scot to associate with the 19 semi-tones of the "post-Guidonian" scale the cycle of 19 solar years, at the end of which the phases of the moon return in the same order and at the same times of the solar years (Meyer 2009). This is known as the "Metonic cycle" (HMNAO and USNO 2013, metonic cycle), a period of almost exactly 19 years after which the lunar phases recur on the same date in the calendar (give or take a day).

Where did Michael get this idea of a 19-tone scale? Naturally, one can think of Muslim sources. The number 19 is of interest in Islam and the B'hai faith which came later in the 19^{th} century (considered a heresy by Muslims). Alternatively, if Michael Scot did indeed come from Northern England or Southern Scotland, one could think of Celtic sources. In Celtic mythology, 19 is a sacred number of the goddess Brigid. According to the ancient Irish text, Giraldus Cambrensis (Burns 1998), there were traditionally 19 priestesses who would tend the sacred flame, one per day for 19 days, by feeding it the sacred wood of the Hawthorn tree. The number 19 was sacred to Brigid because, according to the "Book of Dunn Cow", it represented the 19-year cycle of the "Great Celtic Year" and the time it takes for the new moon to coincide with the Winter Solstice from one cycle to another.

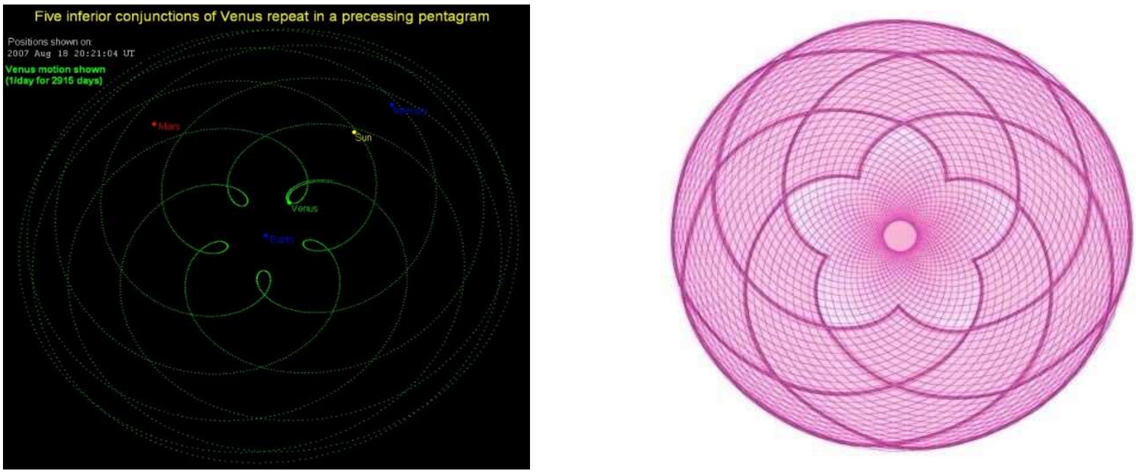
What could have inspired both Ibn Arabī and Michael Scot? Thorndike suggested the Arabīc translation of Pseudo-Aristotle's "The Secret of Secrets" also played a role in introducing Pythagorean musical theories to Islamic philosophy and culture (Rašić 2022, n. 4). This work led Al-Kindi (d. 873) (O'Connor and Robertson 1999) to the conclusion that "all forms are ruled by supercelestial forms through the spirits of the spheres" and that all incantations and images receive their power from the heavenly spheres. (Thorndike 1922, 236, 240). The "Secret of Secrets" was a treatise purporting to be a letter from Aristotle to his student Alexander the Great on an encyclopedic range of topics, including statecraft, ethics, physiognomy, astrology, alchemy, magic, and medicine. A first translation of this work from Arabīc into Latin was done by John of Seville around 1120 CE and a second translation by the canon Philip of Tripoli around 1232 CE. Himself a translator, Michael Scot would have been aware of them.

Scot also considers why ordinary people cannot hear the music from the moving astronomical bodies but his speculation considers physical rather spiritual reasons. Amongst the considered possibilities, the huge distance between the Earth these bodies, the possibility that the Moon actually blocks the effects of the more distant planets and interestingly the "thickness of the air" (Meyer 2009, 125). At any rate, no matter how Michael Scot got these ideas linking music to astronomy, whether from Muslim sources such as Ibn

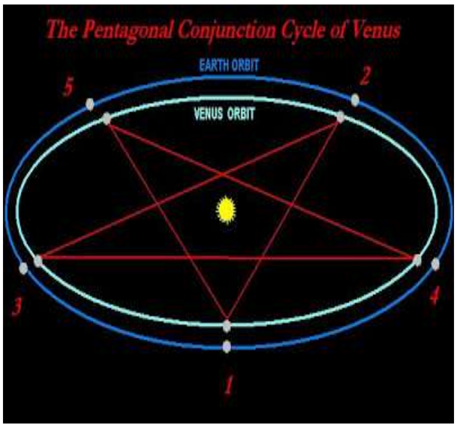
Arabī or others, it is likely that Kepler is tributary to Michael Scot and his unique re-introduction of the "Music of the spheres" into Europe.

Example of Venus

(a) Traced Orbit (b) Pattern



Depiction of the Earth: Venus 8:13 near resonance. With Earth held stationary at the center of a non rotation frame, the successive inferior "conjunctions" (HMNAO and USNO 2013, conjunction) of Venus over eight Earth years trace a five-petal "Rose of Venus" flower pattern leading to Figure 2. - Courtesy Wikipedia.

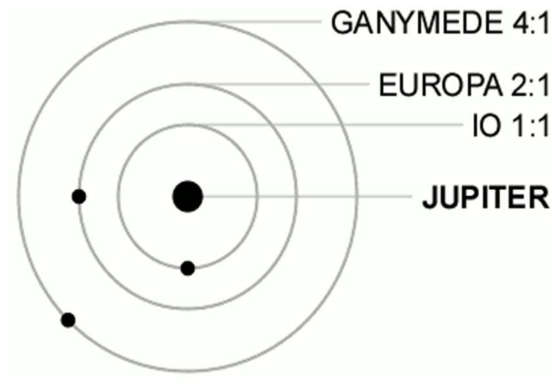


Pentagram traced by the successive inferior conjunctions of Venus over eight Earth years (reflecting the difference between the numbers in the ratio). The pentagram embeds the golden number (T. C. Scott and Marketos 2014b). - Courtesy Indras Net.

Venus provides us with a fascinating example of orbital close commensurability involving Fibonacci numbers. The Earth orbits the Sun once every 365.256 days while Venus orbits the Sun once every 224.701 days. So, the ratio is $224.701/365.256 \approx 0.61518624$, which is only 0.032% away from exactly $8/13$ (or written as $8:13$). Thus, the orbits of Earth and Venus arrive at almost the same configuration after 8 Earth orbits and 13 Venus orbits. (The mismatch after 8 years is only $1.5°$ of Venus' orbital movement.) Both planets approach each other and reach an inferior conjunction in "synodic periods" (HMNAO and USNO 2013, synodic period) of 584 days, on average (Williams 2023). As it does so, it changes from the "Evening Star", visible after sunset, to the "Morning Star", visible before sunrise.

The ratios of consecutive Fibonacci numbers give the best approximations to the golden ratio $\phi = (\sqrt{5} - 1)/2 \approx 0.618034 \ldots$ [not to be confused with the standard golden number $\Phi = (\sqrt{5} + 1)/2 = 1/\phi$]. This number plays a role in celestial mechanics: the Kolmogorov-Arnold-Moser theorem (Tabor 1989) says that two systems vibrating with frequencies having a ratio equal to ϕ are especially stable against disruption by resonances, because this number is hard to approximate well by rationals. However, the Venus/Earth period ratio 0.615187 is actually closer to the fraction $8/13 = 0.61538461 \ldots$ than ϕ.

Orbital Resonances



The three-body Laplace resonance exhibited by three of Jupiter's Galilean moons. This diagram is not to scale. Courtesy Wikipedia

In the Jupiter system, the orbital periods of the Galilean moons (Barnes 2011) Io, Europa and Ganymede are almost exactly in the ratio $1:2:4$ as illustrated in Figure 3, and this magnifies the mutual gravitational forces that the moons exert upon one another. Perhaps counter-intuitively, this actually stabilizes the system, as the orbits become "locked" in a configuration which minimizes the mutual interactions. It is impossible for all three satellites to reach their closest approach to one another simultaneously. When any pair of satellites are at their closest, the third is in a distant part of its orbit relative to the other two. This was first studied by the great French mathematician and astronomer Laplace, and is named the Laplacian resonance in his honor (Barnes 2011).

The same situation exists between pairs of moons of Saturn. Mimas and Tethys have orbital periods near the ratio $1:2$, as do Enceladus and Dione, even though these pairings are not between nearest neighbors. Further from Saturn, Titan and Hyperion have orbital periods near the ratio $3:4$. In each case, this induces a "libration" (HMNAO and USNO 2013, libration), or large periodic oscillation of long period, in the along-orbit position of the satellites. In order to correctly calculate the position of Mimas in its orbit, for example, one has to add a periodic term with amplitude of 43 degrees and a period of 71 years to its orbital longitude (Harper and Taylor 1993). Mimas itself orbits Saturn in less than one Earth day. In the case of Hyperion, whose orbit is markedly eccentric, extreme close approaches to Titan, the largest moon of Saturn, are prevented by the fact that when the two moons have the same orbital longitude, Hyperion is always at the "apocenter" (HMNAO and USNO 2013, apocenter) of its orbit, and thus as far from Titan as its orbit allows it to be.

Among the planets themselves, no such resonances currently exist, although there's evidence that in the very early history of the Solar System, the orbits of Jupiter and Saturn became locked into a strong resonance which forced the orbit of Saturn to expand in lock step with Jupiter. Today, the orbital periods of the two planets are close to the ratio 2: 5, and this amplifies the gravitational effect that they exert on one another. Whilst the orbits are not "locked" together, an accurate calculation of the position of Saturn in its orbit must include a periodic term with an amplitude of 0.8 degrees and a period of 880 years. This is known historically as the Great Inequality. (0.8 degrees might not seem much, but it's the distance that Saturn travels along its orbit in about a month, so it is significant.)

Resonances have long fascinated celestial mechanicians, because they are associated with chaotic dynamics. In particular, chaotic orbits result from overlapping resonances (Lecar et al. 2001). An interesting recent paper (Zink, Batygin, and Adams 2020) suggests that in the far distant future, the entire Solar System may disintegrate after the orbits of Jupiter and Saturn evolve into a true 5: 2 resonance, and then encounters with passing stars tip the Solar System into a chaotic region of that resonance.

As astronomers discover greater numbers of exoplanetary systems, some of them show evidence of multiple resonances between planets in a system, like the Io-Europa-Ganymede configuration. A remarkable recent example (Leleu et al 2021) is the discovery, using the Transiting Exoplanet Survey Satellite, of a star with six planets in a chain of resonances with periods in the ratios 2:4:6:9:12. Leleu et al note that "our current understanding of planetary system formation theory implies that such configurations are a common outcome of protoplanetary discs". A lot of theoretical work is being done to complement the observational research, mainly using large-scale numerical simulations of planetary systems with multiple resonances, to try to understand how such systems evolve, and whether they are stable on timescales equal to the lifetime of the parent star.

Fibonacci numbers abound. E.g. Aschwanden *et al.* found that out of 5454 exoplanets, "the most prevailing harmonic ratios are 2: 1, 3: 2, and 5: 3 in 73% of the cases" (Aschwanden and Scholkmann 2017).

The book "Solar System Dynamics" by Carl Murray and Stanley Dermott (Murray and Dermott 1999) is a very good introduction to resonances and chaos in planetary systems. Chapters 8 and 9 of this book provide an excellent introduction to the subject, including the discovery in the 1980s that a planetary (or satellite) system can become chaotic when there are regions of phase space where resonances overlap. Orbital resonances are an extensive subject in astronomy. These authors have a long list of references to some major pioneers like Jack Wisdom (Wisdom et al. 2022), Martin Duncan (Lepage and Duncan 2004), Thomas R. Quinn (Barnes et al. 2015) and Scott Tremaine (Tremaine 2020).

Conclusions

Kepler's earlier model based on platonic solids to model the distance proportions between the planets has been discarded especially in the light of more precise data. However, his attempt at associating the Music of the Spheres with planetary orbits would lead to the discovery of orbital resonances in e.g. the moons of Jupiter and Saturn as well as many exoplanets. From Pythagoras to Ibn Arabī to Michael Scot to Kepler, we have followed an amazing and nearly continuous thread linking ancient astrology to music eventually leading to modern astronomy. In this particular international trajectory, Michael Scot played a key role in transmitting the knowledge of Ibn Arabī and other Muslim sources to medieval Europe.

In hindsight, it should not be too surprising that this alleged connection between music and astronomy finally succeeded. Thanks to modern Physics, in particular quantum mechanics, namely as expressed by Planck's law, energy is directly proportional to frequency and frequency is a measure of any cyclic motion, like the motion governing planets of our solar system as well as the very harmonics that make up sound and music.

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