transversal
International Journal for the Historiography of Science

Transversal: International Journal for the Historiography of Science, 2023 (15): 1-13
ISSN 2526-2270
Belo Horizonte – MG / Brazil

Article

Stylistic Approach to the Brachistochrone Problem

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Abstract:

The notion of style is frequently used, and in some instances, without the necessary rigor. Authors such as Crombie, Hacking, Bueno and Granger consider presenting a general concept to be essential and sufficient to grasp the notion of style. They found a possibility to apply a strict concept of style even to science and mathematics. My objective in this article is to test, using a fundamental criterion raised by Bueno (2012), the possibility to characterize a mathematical local style from a particular event in the history of mathematics: the Brachistochrone problem. Because this very problem has different solutions, which allows them to be analyzed in order to verify an occurrence of style on their mathematical development. Moreover, Bueno offers a criterion that establishes a minimal unity of structure to a notion of style, even in mathematics. There are two important problems that any concept of style should face: (i) the impregnation problem posed by Bueno; and (ii) the cognitive relevance proposed by Mancosu. The former presents a serious implication in supporting a proper style in mathematics, because any mathematical object needs a preceding mathematical theory that characterizes it; and if it is not possible to constitute a style in mathematics, then recognizing its cognitive relevance could also be compromised.

Keywords: Concepts of Style; Brachistochrone Problem; History and Philosophy of Mathematics.

Received: April 23, 2023. Reviewed: November 03, 2023. Accepted: December 02, 2023.
DOI: http://dx.doi.org/10.24117/2526-2270.2023.i15.10

Concepts of style are often used in many different cases, but there are two major meanings attributed to it: (i) individual features that indicate a unique bond between an author and his/her work; and (ii) a collective feature that characterizes a particular activity in a certain domain. It is important for philosophy to refine this notion in order to better understand (or maybe conceive) a proper concept of style. Moreover, philosophy should discuss and (if possible) indicate ways to an epistemology of style. Mancosu (2017) synthesized such need in a simple question: does style have cognitive relevance or does it not? In this paper, I will first present and discuss a number of key concepts of style. Three of them are interconnected, which are Crombie's style of thinking, Hacking's style of reasoning, and Bueno's narrow style of reasoning, and the other one is Granger's general stylistic analysis.

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Then, I will present the Brachistochrone problem as an example in the history of mathematics in order to evaluate if it is possible for mathematics to own its proper style or not – my guideline will be Bueno's five fundamental components as a criterion that indicates the very presence of a style. Finally, I will present a perspective regarding the results of such criteria for a style of mathematics.

Narrow style of reasoning

We usually use the term *style* in an ordinary sense as a particular way to do or to be something. In the Arts, this term is also a characteristic designated to a group of people or to a particular period. Scholars have explored these two meanings in order to give them some philosophical or (and) historical background. Otavio Bueno (2012) is one of those scholars, and I am going to use his concept of style based on five fundamental components as a criterion, which indicates what could or could not be considered a style, in order to better understand what that term ultimately means in mathematics. This criterion fixates a spot higher than the theory[2] of that domain, to which some style is alluded. None of these five items depends on any specific theory at all of a certain domain other than style ones. Bueno's five basic components (shown below) helped to characterize *style* as a form of investigation:[3]

(i) identify questions;
(ii) afford techniques and procedures to answer those questions;
(iii) own valid patterns of inference to investigate objects in a certain domain;
(iv) employ heuristic resources and
(v) identify constituting conditions to certain objects.

We can consider an example to understand how these five components could circumscribe an appropriate style, independently of any scientific theory for the production of inferences. The example, developed by Bueno about George Palade, shows us how Palade performed procedures in his laboratory, using different measurements and analyses until he declared consistently that an unknown scientific object was detected in the cell's biology. Forward analysis concluded that these unknown cellular components were composed mainly of ribonucleic acid, and were thus called ribosomes. The scientific community accepted the existence of this new cellular compound, the ribosome, under the influence of visual culture,[4] after seeing the images produced by the Transmission Electron Microscope (TEM). Today, ribosomes are studied in a field of research within molecular biology that covers protein synthesis and the important role played by ribosomes in this process.

This example illustrates a specific style developed by Bueno, a narrow style of reasoning or, precisely, the instrumental style, which involves the study of cell structure and behavior, based on the outputs of imaging instruments. The micrographs produced by TEM provide visual evidence for the existence of relevant structures: the ribosomes. Certain marks presented on the surface of these micrographs are interpreted as evidence of the presence of the corresponding objects within the sample. The inferential devices used in this case were the images, and they allowed one to infer, because of the reliability of TEMs, the presence of the given phenomenon from suitable traits in the micrographs.

> Styles of reasoning play a significant role in shaping our understanding of scientific activity (…) This is, in part, due to the role they play in constituting that activity. As I use the concept, a style of reasoning [narrowly understood] is a pattern of inferential

[2] A body of principles offered to explain something.
[3] To make questions in a certain domain.
[4] To create, to manipulate, and to disseminate images in scientific practice.

> relations that are used to select, interpret, and support evidence for certain results. If we consider different domains of scientific research, different styles of reasoning are often involved. (Bueno 2012, 657)

We have seen a narrowly understood instrumental style of reasoning at work, but it is important to certify if this new instrumental style conforms to Bueno's five basic components. In order to do so, we can apply the five fundamental components of a narrowly understood style to the example above:

(i) **identify questions**: examine data from new scientific instruments and seek to create inferences from possibilities;
(ii) **afford techniques and procedures to answer those questions**: employ scientific instruments including computational resources;
(iii) **own valid patterns of inference to investigate objects in a certain domain**: "involve (deductive) logic (which may or may not be made explicit) and, more broadly, suitable information transfer procedures, which are highly context sensitive and rely on additional assumptions about the domain under consideration" (Bueno 2012, 660);
(iv) **employ heuristic resources**: use images produced by scientific instruments, auxiliary hypothesis and triangulation techniques, and
(v) **identify constituting conditions to certain objects**: constitute scientific objects through certain characteristics and empirical results.

Thus, the narrowly understood instrumental style of reasoning is, according to Bueno, in conformity with the five basic components of a concept of style because the criteria based on those fundamental components were successfully applied, grasping the minimal structure that a style should have.

Narrow styles of reasoning are also applicable in mathematics, and they are in accordance to principles that describe a certain class of objects and relations between them. Connections are established between relevant objects in order to determine their properties and also to determine other relations that those objects bear in respect to others within the same domain. The inferential mechanisms used in mathematics, besides logic, which plays a limited role in mathematical practice, are: diagrams, drawings, pictures, mental images, and geometrical interpretations. Many other visual devices also play an inferential role, according to Bueno (2012, 661).

Bueno is offering a particular position from within a broader concept of style, which was conceived by Crombie and philosophically refined by Hacking. For a better understanding of what Bueno means with the narrowly understood style of reasoning, it is important to emphasize that this style is fundamentally an *inferential* one in its nature. Such style allows for the inference of important information in respect to the domain under investigation, as in the example above about the discovery of the ribosome.

On the one hand, narrowly understood styles of reasoning provide more specific information about some particular domain of inquiry without losing generality. On the other hand, the broader styles of reasoning from Crombie and Hacking neglect, in science and also in mathematics, refined features that lie upon a practice-oriented framework because these styles consider (to some extent) a general concept of style that puts science and mathematics on a specific *plateau*. One of those *plateaux,* of which mathematics is part *par excellence,* is Crombie's style of thinking, which was commented by Bueno:

> If we are interested in making sense of postulational reasoning in mathematics, it is crucial to examine the relevant style of reasoning at a substantially lower level of abstraction, so that significant differences within postulational reasoning can be examined and assessed. In contrast, narrowly understood styles of reasoning –

> henceforth, 'narrow styles of reasoning' – operate at a level of abstraction that allows for the identification and study of these differences, while still preserving some generality. (Bueno 2012, 659)

Hence, narrow styles of reasoning are much more properly suitable to the diversity in mathematical practice than Crombie's concept of style supported by Hacking.

Style of Thinking and Style of Reasoning

Alistar Cameron Crombie conceived a monumental work – Styles of Scientific Thinking in the European Tradition (1994) – regarding European science history, in which he proposed a historical, analytical interpretation of long periods of time. To that end, an array of concepts (related to philosophy of knowledge and its objects, nature and its science, arguments and its evidences) gives birth to a European mindset that persists in time and resists within its society and culture. A style of thinking is, ultimately, related to commitments. For example, the ancient Greeks introduced a form of thinking to European rationality based on two commitments: natural causality and formal proof.

From a historical analysis to European-thinking comparative anthropology, Crombie (1995) considers the existence of some structures that persist in time, deeply rooted in commitments or dispositions. The author considers two different moral and intellectual commitments: one concerning nature and its perception, and the other concerning science and its investigation, problems/solutions, explanations, arguments, language[5] and even some errors – i.e., everything that is related to styles.

> Within these general commitments, scientific thinking became diversified into a number of different styles of inquiry, demonstration and explanation, of which I have identified a taxonomy of six. The novelty was in the style. It is illuminating to focus on the critical occasions of intellectual orientation, leading to the maturity of each style. There is a logical and a chronological sequence, in each arose in a cultural context where an assembly of different but cognate subject-matters, scientific, artistic, economic, and so on, was united under a common form of argument (…) A scientific style, with its commitments, identify certain regularities in nature, which became the object of its inquiry, and defined its questions, methods and kinds of evidence appropriate to acceptable answers within that style. (Crombie 1995, 234)

The six styles raised by Crombie are:

1. The postulation style: based on mathematical and logical argumentation, it consists of deductive proofs from explicit principles (e.g., Euclidian geometry, Aristotelian syllogisms);
2. The experimental style: it is related to the control of postulates and the search for new ones with the help of observation and measurement;
3. The hypothetical modeling style: it consists of conceiving models to explain properties of unknown phenomena;
4. The taxonomic style: it refers to the organization, ordination and comparison of phenomena involving groups or populations;
5. The probabilistic and statistical style: it concerns the analysis of regularities in events related to manifold populations or groups of individuals;

[5] Crombie considers that language embodies a theory of meaning, logic, a classification of experience, a sentient conception, a knower (and an agent) and his/her objects, and an apprehension of space-time existence.

6. The historical-genetic style: it combines analysis and synthesis of genetic development by observing present regularities to infer the past (and the present would be explained by developments from the past).

One should ask if the postulation style is indeed a style and, according to Bueno (2012), the five fundamental components should answer this question affirmatively if the style in case is in conformity with those components. Thus, one can find the following:

(i) **identify questions**: examine what follows from first principles or postulations;
(ii) **afford techniques and procedures to answer those questions**: employ definitions and mathematical proof in order to establish results;
(iii) **own valid patterns of inference to investigate objects in a certain domain**: adopt the classic deductive logic as an inferential pattern;
(iv) **employ heuristic resources**: use diagrams; and
(v) **identify constituting conditions to certain objects**: constitute objects by means of postulations and derivations regarding the same objects.

Apparently, the postulation style is in conformity with Bueno's criterion: the narrowly understood instrumental style of reasoning. Nevertheless, the fifth basic component is not properly adequate to mathematics as it is to science, since mathematical objects are inexorably characterized by mathematical concepts, thus making a genuine mathematical style unfeasible. Therefore, for every attempt to conceive a mathematical style, when its objects are characterized, a mathematical concept will be necessarily invoked. That way, the mathematical plasticity makes impossible for us to consider a proper mathematical style. This difficulty was called, by Bueno, the impregnation problem.

Hence, to complete the triad of concepts of style related to each other, before presenting a totally different one (Granger's style), Hacking's concept will be presented to adjoin Crombie's and Bueno's concepts.

Ian Hacking (1992) supports the addition of two new styles to Crombie's list, namely: the laboratory style, and the algorithmic style. The former was composed from the combination of two of Crombie's styles: the experimental style and the hypothetical modeling style. The latter was created outside the European circle, and in fact its origin refers back to the Medieval Islamic world (Whinther 2012, 596). Hence, to avoid psychological, subjective or relative interpretation to his concept of style, Hacking changed from style of thinking to style of reasoning.

According to Hacking, there are no conditions that should be considered sufficient and necessary for something to be a style, and only some key features should be selected. A new style introduces new objects, new kinds of sentences and new methods of reasoning. "For example, with the mathematical style comes new abstract mathematical objects, a new method of proof and new kinds of sentences expressing axioms and theorems." (*ibid.*) The emergence of these elements converges to a style itself.

Hacking, differently from Crombie, highlights discontinuities in the development of styles. However, he points out that the concept of style crystallized in history and thus fixated how to proceed in the future based on initial precursors, just like Crombie did. Hacking supports two other controversial characterizations of style: autonomy and self-authentication. Style autonomy relies on its independence of its cultural and environmental origins (e.g., the independency between the postulation style and Ancient Greece), which means that styles do function well in different social and cultural contexts after crystallization. Style self-authentication means that styles of reasoning make relevant kinds of sentences that are candidates to be true and false, and thus styles do not need outside reasons to support or to justify them.

General Stylistic Analysis

Gilles-Gaston Granger (1974) establishes a relation between form and content as a process of working from a historical domain, thus producing some intellectual work that is presented by a social and historical practice. Individuality is opposed to structure but, in spite of it, the living practice and its elements are incorporated to the form as non-casual redundancies or overdeterminations. His concern is to develop a general stylistic analysis in which every practice has an inseparable style, and in which both – practice and style – are concretely presented as a factor of style[6]. Granger based his general stylistic analysis on the Kantian transcendental aesthetics; i.e., every formal condition of knowledge determines a type of objectivity *a priori*. Moreover, he seeks for general conditions to structure insertions within an individual practice and, to this end, he analyzes mathematical works provided by historical subjects, aiming at giving them form and focusing on more general conditions of practice. In mathematics, practice arises from a certain form, which has to be adequate for different possibilities. Mathematical redundancy escapes from a constituted framework, and thus it does not have meaning (to some extent it is just a non-explored residue). A mathematician can present, in the mathematical experience, different successful manners to conceive one and the same form, or different mathematicians can present varied compositions of an identical form in each mathematical experience.

> Style appears to us here on the one hand as a way of introducing the concepts of a theory, of connecting them, of unifying them; and on the other hand, as a way of delimiting what intuition contributes to the determination of these concepts. (Granger 1974, 30)

The example that Granger uses to support his concept of style is based on three different forms of presenting complex numbers, and it establishes different styles for each of them. I will use only one of these forms against his own concept in order to show that Granger is not free from the problem of theory impregnation.

Two parts compose the trigonometric form of the complex number, which are the *modulus* (ρ) and the *angle* (ϑ). **Figure 1** presents the trigonometric form and refers to an extension and rotation operator applied to vectors. It also intuitively suggests, according to Granger (1974): the multiplication rule[7]; the transformation from polar to Cartesian coordinate system; and the additive properties of the complex number in the Cartesian form.

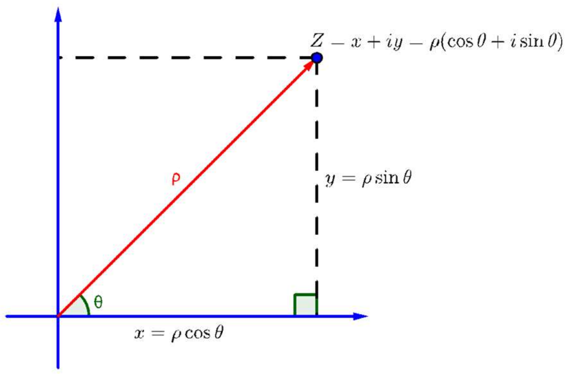

Figure 1: Polar coordinate graphic for complex numbers

[6] A meaningful contact between a structure and a live circumstance that was experienced.
[7] Multiply the *modulus* and sum the *angles*.

It is time to recall that intuitions neglect mediation, hence if we intuit something - like the multiplication rule regarding complex numbers at a polar coordinate system - then we will do so without any kind of help. It seems to me that it is very unreasonable to consider that the multiplication rule of complex numbers in polar coordinate systems does not need, however minimum, previously understood concepts, which gives basis to vectors, operations with vectors, plane geometry of circles, Argand-Gauss plane, etc. Also, how are these concepts related to each other in order to conceive the multiplication rule of complex numbers in polar coordinate systems? Granger used mathematical theory to compose a style in mathematics and, whenever he tried to refer to the complex number (the mathematical object) illustrated in the Cartesian plane according to the polar system, some conceptual characteristic from vectors was needed. No intuitions were evoked, only concepts related to vector theory, and that is why his concept of style failed to pass the impregnation problem conceived by Bueno.

The Brachistochrone Problem

It is important to test the possibility of either having or not having style in mathematics in a historical episode. To do so, I believe that the Brachistochrone[8] problem could help us in the analysis of mathematics in order to conclude whether it is possible to sustain a concept of style in it. I am going to apply the criterion of the five basic components to the narrow style of reasoning in mathematics, which is similar to what Bueno did in his ribosome micrograph example based on George Palade's research. In other words, I am proposing a mathematical episode, the Brachistochrone problem, with the aim of verifying the very possibility to sustain the narrow style of reasoning in mathematics.

The Brachistochrone problem is crucial to my purpose because it has united different mathematicians facing the same challenge, each one – with their theories, abilities and practices – confronting it and achieving the same solution: the ordinary cycloid. Initially, Galileo was the first mathematician to work with this problem (to find the fastest curve between two given points), but he placed it in the theorem 22 (proposition 36) scholium of his *Discorsi*[9] (1638). Galileo reached the following result: the circle arch is the fastest curve between two given points, but he committed a mistake that deviated him from the right curve, the ordinary cycloid. (At that time, it is important to emphasize, Galileo already knew the cycloid, its constructing characteristics and the proportion of its surface, but he did not know that it was the fastest curve between two given points.)

The Brachistochrone problem is the following: consider two points not in line on a perpendicular plane, vertical in relation to the horizon. Under the action of gravity, a body slides from the higher to the lower point; the problem requests the curve that connects these two points, in which the body would travel in the minimum amount of time possible (**Figure 2**). Johann Bernoulli had proposed it to his contemporary mathematicians in order to claim the superiority of the Leibnizian theory and practices. He proclaimed it publicly for the first time in the *Acta Eruditorum* (1696) as *Problema Novum.*[10]

[8] The problem to find the fastest curve, "Brachistochrone" comes from the connection of two other terms in Greek: βραχιστος (shortest) and χρόνος (time).
[9] The title abbreviation of *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*.
[10] Johann Bernoulli also used the Brachistochrone problem in letters to Leibniz (2011, 49).

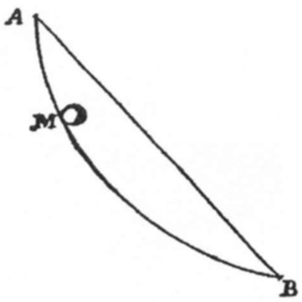

Figure 2: Figure 5, table 5, *Acta Eruditorum*, June 1696

Johann Bernoulli and Leibniz decided to extend the deadline for presenting the solutions until the following Easter, so that mathematicians outside Germany – mainly from France, Italy and the Netherlands – could have enough time to dedicate themselves to it. In May 1697, Johann Bernoulli published all the solutions he found in the *Acta Eruditorum*. Leibniz (1697), Johann (1697) and his brother Jakob Bernoulli (1697) were the first to present a correct solution, and later Marquis de l'Hospital (1697) also rightly answered the problem. Finally, Johann Bernoulli recognized[11] Newton's solution published anonymously on *Philosophical Transactions* in January 1697 and, because of that, he republished it as another solution presented to the Brachistochrone problem in the *Acta Eruditorum* (1697).

Each solution presented in May 1697 in the *Acta Eruditorum* has an impressive diversity in its mathematical arguments, even though all of them reached the ordinary cycloid as the fastest curve. I will consider three of those solutions: the ones by Leibniz, Johann Bernoulli and Newton, mainly in terms of their mathematical arguments. My purpose is to apply the narrow style of reasoning concept to them. I will analyze the corresponding features of that concept of style in these historical instances in order to verify whether or not there is a consistent stylistic unity among them.

Leibniz (1697), on the one hand, first considered the practice of presenting problems in mathematics the best way to put brilliant minds at work. Then, he exalted his theories and methods and Johann Bernoulli's mathematical abilities and generosity to let others exercise themselves in such an attractive and challenging problem. Afterwards, Leibniz described the *problema novum*, presenting it in details. Next, he presented the mathematicians who answered it (Bernoulli brothers, l'Hospital and Newton). Later, he explained that Johann Bernoulli only presented his indirect answer rather than his direct ones because of former considerations to the dioptric phenomenon. Then, Leibniz considered that the Brachistochrone problem, presented in the way it was, promoted a new employment of the maxima and minima concepts to search for the fastest curve. According to Leibniz (1697, 204), Johann Bernoulli's indirect solution has two major consequences: (i) it determines the continuum curvature of the ray of lights and (ii) it finds the curve that describes its reflexes. Finally Leibniz presented the cycloid as the fastest curve, using its constructing $LM = \widehat{LK}$ property.

[11] Johann Bernoulli (1742, 196) recognized Newton's solution to the Brachistochrone problem by *ex ungue leonem* (i.e., one may judge the lion from its claw).

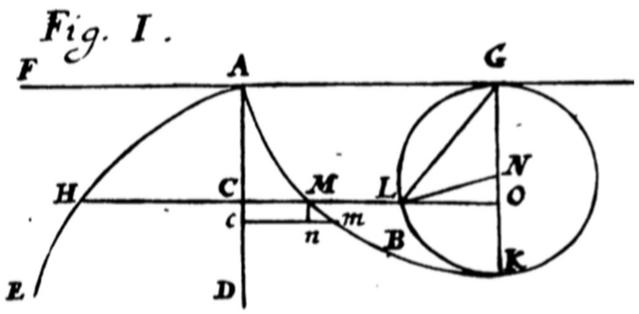

Figure 3: Figure 1, table 4, *Acta Eruditorum*, May 1697

Johann Bernoulli (1697), in turn, first considered the similarity between the fastest curve and the ray of light's trajectory through a non-uniform medium, based on the light property of traversing any medium in the minimal amount of time.[12] Then, he declared that even though there were many methods to determine maximum and minimum, none of them established any subtle connection to the Brachistochrone problem, even the ones inherited from mathematicians before him (like Descartes and Fermat). Later, he declared that he wished to present a particular (or indirect) method rather than a general (or direct) one to solve the problem, and he reinforced that one of his objectives in posing such a problem was to attract mathematicians, like Leibniz did. Afterwards, he exalted Huygens for being the first one to determine the cycloid's isochrone property (proposition 25, *Horologium Oscilatorium*) (Huygens 1986, 69-70), that is, Huygens' famous Tautochrone. (Johann Bernoulli was amazed when he realized that the Brachistochrone and the Tautochrone are the same curve, namely, the cycloid.) Johann Bernoulli presented his indirect method based on the dioptric model (**Figure 3**), that is the curvature of the ray of light, which traverses a non-uniform medium. He considers that, if the law of refraction[13] responds satisfactorily to a kinematic phenomenon, then it does not matter whether they are light or heavy bodies – what really matters is the minimum amount of time, the necessary condition applied in both cases. After that, Johann Bernoulli developed his solution, which is based on the equality between the cycloid differential equation achieved

$$dy = dx\sqrt{\frac{x}{a-x}} \qquad \text{(Eq. 1)}$$

and the cycloid constructing $LM = \widehat{LK}$ property[12]. Finally he presented the cycloid's constructing proportion (Eq. 2), which expresses its fundamental characteristic[14] (**Figure 4**).

$$\frac{AR}{AB} = \frac{AS}{AL} \qquad \text{(Eq. 2)}$$

[12] Fermat (1891, 173) asserts that Nature always acts in an easier and faster way.
[13] According to the law of refraction (or Snell-Descartes' law), the sine of the angle of incidence over the speed of light medium traversing is constant and larger than one (sin α v=const.>1).
[14] The proportion is valid because the angular speeds of the points *R* and *B* are equal, when their generator circles roll at the horizontal *ASL* (Freguglia and Gianquita, 2016, 44-45).

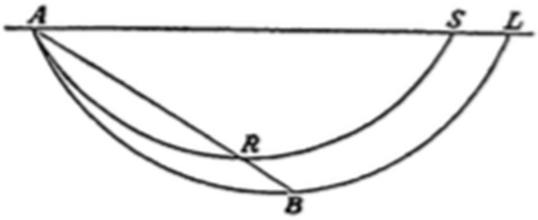

Figure 4: Figure 2, table 4, *Acta Eruditorum*, May 1697

Newton, on the other hand, presented a constructive proportion to the cycloid as the solution to the Brachistochrone problem, which is related to a proportion based on different lengths between two cycloids and the radii from their circle generators. Indeed, it is the same proportion[14] reached by Johann Bernoulli (Eq. 2).

> From the given point A let there be drawn an unlimited straight line *APCZ* parallel to the horizontal one, and on it let there be described an arbitrary cycloid *AQP* meeting the straight line *AB* (assumed drawn and produced if necessary) in the point *Q*, and further a second cycloid ADC whose base and height are to the base and height of the former as *AB* to *AQ* respectively. This last cycloid will pass through the point *B*, and it will be that curve along which a weight, by the force of its gravity, shall descend most swiftly from the point *A* to the point *B*. Q.E.F. (Newton 1967, 226)

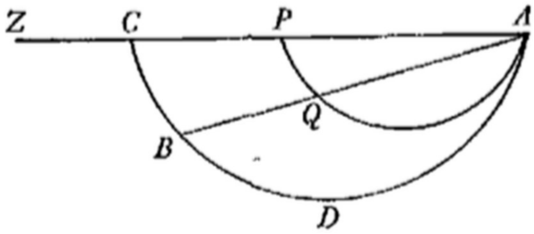

Figure 5: Figure 12, table 4, Acta Eruditorum, May 1697

In short, our conclusion so far has been: (i) that Leibniz supported Johann Bernoulli's indirect method expressed by his optical approach to the Brachistochrone problem and the Leibnizian calculus used to solve it; and (ii) that Newton presented a cycloid fundamental proportion as a justification to this curve in response to the problem, a geometrical fundamental argument that uniquely points out the cycloid as the searched curve. After presenting those three solutions to the Brachistochrone problem, I would like to return to the concepts of style considered in this text, mainly the narrow style of reasoning, in order to analyze this mathematical problem in light of those concepts.

Stylistic Approach

Leibniz, Johann Bernoulli and Newton presented different ways to approach the same problem. It is definitely not a case of different styles applied to the same problem to find the curve that answers it. More precisely, Newton used a different theory[15] (known as method

[15] It becomes clear when later, in 1700, David Gregory asks Newton to develop a mathematical argument to the Brachistochrone problem that is much clearer than the one presented by Nicolas Fatio de Duillier in his *Lineae Brevissimi Descensus Investigatio Geometrica Duplex* (1699); the Newtonian mathematical argument was preserved in *Newtoni Investigatio Curvae Celerrimi Descensus* (ULC.Add.3968.41:2^r).

of fluxion) from Leibniz's[16] and Johann Bernoulli's ones (calculus of differences). Thus, it is not possible, as Granger tried to argue, that these three mathematicians expressed their mathematics differently because they used different styles, since they tried different theories to solve the same problem.

As we are dealing with a historical episode of mathematics, it seems more reasonable to consider Bueno's style of reasoning, narrowly understood, because I am proposing to look at specific mathematical practices (i.e., some solutions to the Brachistochrone problem). My concern is to consider a local style rather than a general one (as Crombie and Hacking did). I will employ the five fundamental components of style, expecting the possibility to verify if a unity is formed in respect to a style that could be properly indicated as mathematical style. This methodology is similar to what Bueno implemented when he was analyzing George Palade's ribosome discovery to argue that in science it is possible to defend an instrumental, narrowly understood style of reasoning.

Firstly, I suggest considering the importance that problems have in mathematics. Leibniz reinforced the role played by problems in mathematics:

> The art of submitting problems to geometers is a generalized practice, and is profitable for everybody, providing it is not done with the intention of bragging about one's own successes, but it is done, on the contrary, with the idea of inciting others to discover; that is, in such a way that discovery is enriched with the particular method where each personality contributes to the art of invention. (Leibniz 2000, 42)

The first component that Bueno considers is precisely the type of question. In our example, we are treating the Brachistochrone problem to create inferences from mathematical possibilities that answer it correctly. Therefore, mathematical internal rules must be followed to prove (generally) a mathematical argument. The second fundamental component is to afford techniques and procedures to answer those questions. We have seen that Johann Bernoulli securely applied the analytical method in his answer, since he considered the known curve, and, of course, the law of refraction had to be inferred; the same law that has incorporated the necessary condition to the minimal amount of time, the Fermat principle. Johann Bernoulli was in the right direction to his solution, since he developed the known cycloid differential equation as a consequence of the employment of the refraction law. At the end, he proceeded to the synthetic method, showing the essential constructing proportion that composes a cycloid (Eq. 2), which is similar to what Newton did. The third fundamental component regards valid patterns of inference to investigate objects in a certain domain. In our example, it is clear that Leibniz, Johann Bernoulli and Newton used deductive logic in a rigorous but informal mathematical argument. The fourth fundamental component concerns heuristic resources: in our example, only Johann Bernoulli presented a heuristic resource, namely the optical approach, which was consistently employed to a kinematic phenomenon, under the same condition: the minimal amount of time. That is, no matter what kind of phenomenon is under analysis (whether optical or kinetic), if the minimal condition is required, the same law is applied, without the need of any scrutiny. At last, the fifth fundamental component is related to identifying constituting conditions to certain objects. According to our case, the cycloid, the answer to the Brachistochrone problem, is characterized either by the cycloid differential equation (Eq. 1) or by the fundamental constructing proportion (Eq. 2), or even by the construction condition (proposition 14, *Horologium Oscilatorium*) (Huygens 1986, 50). Hence, Bueno's five fundamental components

[16] Leibniz has an unpublished answer to the Brachistochrone problem, in which he employs his calculus of differences to reach the correct curve, proving the cycloid differential equation. (Goldstine 1980, 35-38)

to a concept of style applied to mathematics, using the Brachistochrone problem as an example, should be synthesized as:

(i) **identify questions:** to create inferences from mathematical possibilities that answer a problem correctly in the mathematical domain;

(ii) **afford techniques and procedures to answer those questions:** to employ valid mathematical methods, procedures, techniques, rules, etc.;

(iii) **own valid patterns of inference to investigate objects in a certain domain:** "involve (deductive) logic (which may or may not be made explicit) and, more broadly, suitable information transfer procedures, which are highly context sensitive and rely on additional assumptions about the domain under consideration" (Bueno 2012, 660);

(iv) **employ heuristic resources:** to use diagrams, analogies, problem solving techniques, etc.; and

(v) **identify constituting conditions to certain objects:** to constitute mathematical objects through certain characteristics that establish it.

Final Considerations

We conclude that mathematics does not incorporate a style. In virtue of a certain mathematical theory is required to constitute a mathematical object. At first, it seems that mathematics affords a notion of style because of the successful applicability of Bueno's criterion, which establishes a minimal unity structure. Nevertheless, in mathematics, those five components are not sufficient to affirm consistently that it embodies a style, since a mathematical object necessarily needs a previous mathematical theory, as mentioned earlier and seen partially in our mathematics historical example. The solutions presented here to the Brachistochrone problem show that, in order to characterize the cycloid, at least one of these properties were necessarily needed: (i) a basic construction condition (proposition 14, *Horologium Oscilatorium*) (Huygens 1986, 50); (ii) a differential equation (Eq. 1); or (iii) a fundamental proportion (Eq. 2). Thus, we can assert that the theory of proportions, Euclidean geometry, and calculus of differences were needed to characterize the cycloid curve. Therefore, Bueno's fifth fundamental condition to a style notion cannot not be fully applied to mathematics. Such impossibility does not allow for the successful characterization of a style in mathematics as it was the case in science, since a scientific object could be constituted using empirical results. Our example, the Brachistochrone problem, concretely indicates this fundamental difficulty.

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